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FLIGHT INVESTIGATION USING VARIABLE-STABILITY AIRPLANES OF MINIMUM STABILITY REQUIREMENTS FOR HIGH-SPEED, HIGH-ALTITUDE VEHICLES

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SUMMARY

The pilot opinion of the flying qualities of vehicles covering a wide range of longitudinal dynamic characteristics has been determined by the use of a variable-stability airplane. Particular emphasis has been placed on determining the minimum level of stability and control characteristics that the pilot can cope with.

There was considerable pilot learning associated with operation in the regions of poor stability characteristics.

In the statically stable region the maximum acceptable value of time to damp to half amplitude of the longitudinal mode for normal operation was about 1 second. For emergency conditions the damping could be reduced to zero over most of the frequency range. The extreme limit of controllability corresponded to a time to double amplitude of the oscillation of about 1-1/2 seconds. In the statically unstable region somewhat shorter times to double amplitude were acceptable to the pilots. The boundary for emergency operation corresponded roughly to time to double amplitude of about 2/3 second and the limit of controllability of about 1/3 second.

Results of brief tests of the limits of controllability in the lateral-directional mode were in fairly close agreement with those of the longitudinal mode.

The pilots found relatively little trouble in handling vehicles with moderate amounts of static instability while they were paying close attention to the control task. However, there was a dangerous situation in that a short distraction of the pilot's attention could allow the unstable vehicle to diverge to the point that it was difficult or impossible to recover.

INTRODUCTION

The provision of adequate control for re-entry vehicles and for high-speed, high-altitude aircraft is a major problem since in the course of a normal flight such vehicles traverse an extremely wide range of dynamic pressures and undergo major changes in their centers of pressure. As a result, the dynamic characteristics of a vehicle vary over a wide range and the basic vehicle may even become unstable. It is now generally believed that some sort of stability augmentation must be used about all three axes in order to provide adequate control. One of the most promising types of augmenters being developed is the adaptive control system in which the over-all response of the vehicle remains nearly constant through the full range of basic stability parameters. While such a control system will be of tremendous help, there are two questions which must be answered before such a system can be perfected: (1) What dynamic characteristics are best from the pilot's viewpoint? and (2) What are the minimum dynamic characteristics a pilot can handle in case of failure of the stability augmentation equipment? The latter question is particularly important because of the large performance penalties due to increased weight and size necessary if adequate basic stability were to be built into most of these vehicles.

Much of the past work (refs. 1 to 8) has dealt with stable vehicles and has been concerned primarily with the first question. This work has been supplemented by attempts to describe the pilot's response characteristics in the form of analytical expressions so that calculations of the over-all stability of the airframe-pilot system could be made (refs. 9 to 11). It is now apparent that more effort is needed to answer the second question. Some recent work on the minimum stability required in the event of stability augmenter failure is discussed in references 12 to 19.

The present report summarizes flight experiences with variable-stability and variable-control-system airplanes operated over a range of vehicle longitudinal and lateral-directional dynamic characteristics. In view of the previous comments, particular emphasis has been placed on the regions of negative stability.

NOTATION

b	basic airplane longitudinal damping parameter $(2\zeta\omega)$, 1/sec
b'	longitudinal damping parameter of equivalent airframe $(2\zeta'\omega')$, 1/sec
b_D'	Dutch-roll damping parameter of equivalent airframe $(2\zeta_D'\omega_D')$, 1/sec
$\bar{c}$	mean aerodynamic chord, ft
F_s	stick force, lb
g	acceleration due to gravity, ft/sec^2
I_X	rolling moment of inertia, slug-ft^2
I_Y	pitching moment of inertia, slug-ft^2
I_Z	yawing moment of inertia, slug-ft^2
I_{XZ}	product of inertia, slug-ft^2
k_D'	square of the equivalent airframe Dutch-roll natural frequency, 1/sec^2
k	square of the basic airplane longitudinal natural frequency, 1/sec^2
k'	square of the equivalent airframe longitudinal natural frequency, 1/sec^2
K_q	pitching velocity feedback gain
K_α	angle-of-attack feedback gain
M	Mach number
m	mass, slugs
n	normal acceleration, g
p	rolling velocity, radians/sec
q	pitching velocity, radians/sec
r	yawing velocity, radians/sec

S wing area, ft^2

s Laplace transform variable

T_2 time to double amplitude, sec

$T_{1/2}$ time to half amplitude, sec

V velocity, ft/sec

v_e equivalent side velocity, $V\sqrt{\sigma}\,\sin\beta \approx V\sqrt{\sigma}\,\beta$

α angle of attack, radians

β angle of sideslip, radians

δ control deflection, radians

δ_a aileron deflection, radians

δ_c command control deflection, radians

δ_r rudder deflection, radians

δ_s stick displacement, in.

ζ_D Dutch-roll damping, fraction of critical

ζ longitudinal short-period damping, fraction of critical

θ angle of pitch, radians

ρ air density, slugs/ft^3

σ ratio of air density at test altitude to that at sea level

σ_1,σ_2 roots of characteristic equation

τ control system first-order time constant, sec

φ angle of bank, radians

ω_D Dutch-roll natural frequency, 1/sec

ω longitudinal short-period natural frequency, 1/sec

L_p rate of change of rolling moment with rolling velocity, per sec

L_r rate of change of rolling moment with yawing velocity, per sec

L_β rate of change of rolling moment with sideslip angle, per sec^2

L_{δ_a} rate of change of rolling moment with aileron deflection, per sec^2

L_{δ_r} rate of change of rolling moment with rudder deflection, per sec^2

N_r rate of change of yawing moment with yawing velocity, per sec

N_{δ_a} rate of change of yawing moment with aileron deflection, per sec^2

A438 N_{δ_r} rate of change of yawing moment with rudder deflection, per sec^2

M_q rate of change of pitching moment with pitching velocity, ft-lb/radian

M_α rate of change of pitching moment with angle of attack, ft-lb/radian

$M_{\dot{\alpha}}$ rate of change of pitching moment with rate of change of angle of attack, ft-lb/radian/sec

M_δ rate of change of pitching moment with stabilizer deflection, ft-lb/radian

Z_α rate of change of normal force with angle of attack, lb/radian

Z_δ rate of change of normal force with stabilizer deflection, lb/radian

$\dfrac{|\varphi|}{|v_e|}$ $\dfrac{|\varphi|}{|\beta|}\dfrac{57.3}{V\sqrt{\sigma}}$ ratio of bank angle amplitude to equivalent side velocity amplitude for the oscillatory mode, $\dfrac{\text{deg}}{\text{ft/sec}}$

$(\dot{\ }),(\ddot{\ })$ first and second derivatives with respect to time

TEST EQUIPMENT

A variable-stability, variable-control-system YF-86D airplane (fig. 1) was used in the longitudinal controllability portion of this investigation. The variable control system was described in detail in references 1 and 2. Since those reports were published, angle of attack and pitching velocity feedbacks have been incorporated in the servo system to provide the airplane with the variable-stability characteristics. A block diagram of the complete system is shown in figure 2. Table I gives the stability derivatives of the basic airplane.

An electronic servo valve replaced the mechanical valve in the alternate hydraulic system on the YF-86D airplane and in this modified system either longitudinal stick force or position signals could command stabilizer motion. In addition, the stabilizer motion is driven in proportion to the angle of attack and to the pitching velocity. The pilot could vary the control system gain, breakout force, and time constant, as well as the angle of attack and pitching velocity feedbacks. The gain between stick force and stabilizer angle could be varied from 1° to 0.04° per pound. At the test conditions of 35,000 feet altitude and 0.80 Mach number and with the basic airplane dynamic characteristics, a range of stick force per g of 1.43 to 35.7 pounds per g could be covered. The breakout force could be varied from 0 to 35 pounds, and the control system "equivalent first-order time constant" could be varied from 0.15 to 4.0 seconds. The angle of attack and pitching velocity feedbacks allowed the stability characteristics to be varied between a short-period frequency of about 8 radians per second and a static divergence condition of a time to double amplitude of 1/4 second. The damping ratio could be varied from about 1.0 to -0.2 of critical.

In the longitudinal controllability tests, stick force commands were used. The stick was disconnected mechanically from the rest of the control system and was restrained by a spring and dashpot system. The approximate stick force to stick displacement transfer function was

$$\frac{\delta_s}{F_s} \approx \frac{0.1}{1 + 0.18s}$$

Figure 3 presents a stick position calibration showing a friction force of about 1.25 pounds. The total stick travel was about 8-1/2 inches.

A variable-stability F-86-E airplane described in reference 6 (fig. 4) was used in the lateral-directional stability portion of the investigation. Typical force deflection calibrations of the rudder and lateral stick movements are presented in figures 5 and 6 showing a friction force of about 20 pounds in the rudder system and about 1 pound in the aileron system.

TESTS

The flight tests for the longitudinal control phase of the investigation were made at 35,000 feet altitude and at a Mach number of 0.80. The stick force of 10 pounds per g was held constant. The resulting maximum $\ddot{\theta}$ response to a step in stick force was calculated and is presented in figure 7. The breakout force was kept at zero and the control system time constant was varied by the pilot. Although the breakout force between the stick force and stabilizer command was zero,

there was about 1 pound of breakout force between stick force and stick displacement, as shown in figure 3. The angle-of-attack and pitching-velocity feedbacks were adjusted to give the desired airframe characteristics – a frequency range of 8 radians per second to a static divergence of a time to double amplitude of 1/4 second and a damping ratio range from 1.0 to -0.22 of critical.

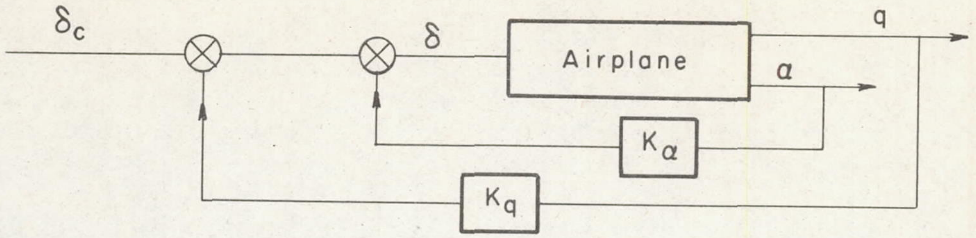
Most of the longitudinal controllability tests were flown by one pilot – an experienced test pilot who had participated in the flight and simulator tests of references 1, 2, 6, 16, and 17. A limited number of flights were made by a second test pilot. In general, the difference in ratings between pilots was less than the scatter in ratings of a single pilot.

The pilots were instructed to rate the flying qualities of the airplane with the minimum control system time constant of 0.15 second using the rating system of reference 20 reprinted here in table II. They were requested then to determine the optimum setting of the time constant and re-rate the system. Finally, they were asked to find the maximum and minimum values of the time constant which would lead to a pilot rating of 5, the boundary between acceptable and unacceptable characteristics (see table II). The ratings were based on the pilot's ability to make rapid turn entries, to stabilize easily in turns, to hold straight and level flight, and to track distant targets.

In the statically unstable flight condition, it was impossible to maintain a stick force gradient of 10 pounds per g (in this condition the stick force per g is negative), so in and near the statically unstable region a fixed ratio of stick force to stabilizer angle of 0.15°/lb was used. In certain conditions the pilots noted that they preferred other settings but not sufficiently to change their ratings. There was no maximum value of $\ddot{\theta}$ in this statically unstable region because $\ddot{\theta}$ increased with time until the system saturated. The initial aircraft response to a step in stick force (assuming zero time lag between stick force and stabilizer angle) was 0.082 radian/sec^2/lb with the stabilizer angle to stick force gain used.

ANALYSIS

To clarify the parameters against which pilot opinion is plotted, it is well to review the function of the variable-stability equipment as installed in the test aircraft. The stability of the airplanes is varied by driving the controls in proportion to various aerodynamic parameters in order to modify the airplanes' response to those parameters. The longitudinal short-period mode (assumed to be a second-order system) can be modified by feedback signals to the stabilizer proportional to the angle of attack and to the pitching velocity as shown in the following sketch:



If the transfer functions of the basic airplane are assumed to be

$$\frac{q}{\delta} = \frac{C_{o_q} + C_{1_q}s}{s^2 + bs + k}$$

and

$$\frac{\alpha}{\delta} = \frac{C_{o_\alpha}}{s^2 + bs + k}$$

then the over-all transfer functions, neglecting servo lags, are

$$\frac{q}{\delta_c} = \frac{C_{o_q} + C_{1_q}s}{s^2 + (b-K_qC_{1_q})s + (k-K_qC_{o_q}-K_\alpha C_{o_\alpha})} = \frac{C_{o_q} + C_{1_q}s}{s^2 + b's + k'}$$

and

$$\frac{\alpha}{\delta_c} = \frac{C_{o_\alpha}}{s^2 + (b-K_qC_{1_q})s + (k-K_qC_{o_q}-K_\alpha C_{o_\alpha})} = \frac{C_{o_\alpha}}{s^2 + b's + k'}$$

where

$$b' = b - K_qC_{1_q}; \qquad k' = k - K_qC_{o_q} - K_\alpha C_{o_\alpha}$$

Thus, through variation of the control system gains, K_q and K_α, the equivalent frequency and damping parameters, k' and b', can be varied over a wide range.

Reduction of Data

Because of lags in the servo system, the equations above do not exactly describe the response of the airplane to stabilizer inputs. It is, therefore, necessary to determine from flight records the actual values of frequency and damping. If the second-order form of the characteristic equation is assumed, the roots of the equation may be determined by the use of the second-order response curves of reference 21 for stable and for oscillatory unstable roots. The method of obtaining the roots in the case of static divergence is shown in appendix A.

RESULTS AND DISCUSSION

Pilot Opinion Boundaries; Longitudinal Mode

Figures 8 and 9 present the pilot evaluations of the control characteristics of the test airplane for dynamic characteristics in the statically stable range. The ratings are given in terms of the pilot rating scheme of table II. The data are presented in figure 8 as functions of the frequency and damping parameters of the characteristic equations describing the airplane's motion, and in figure 9 the faired curves of figure 8 are presented as functions of the frequency and damping ratio. In these tests the stick force per g was maintained at approximately 10 lb/g.

In and near the statically unstable region where it was impossible to maintain a positive stick force per g, a constant stick to stabilizer gain of 0.15°/lb was used. These results are presented in figure 10.

Statically stable region.- It is noticed in figure 8 that in the low and negative damping region (small or negative b'), there is considerable scatter with most of the data points lying to the left of the faired curves. Most of the data in this region were obtained after the pilot had flown the airplane in the region of static instability. It appeared that the pilot's experience in the difficult statically unstable region led him to be more lenient in his ratings in the statically stable regions. Therefore, when the data were faired, more weight was placed on the less lenient data obtained before flights in the statically unstable region.

The effects of modifying the control system time constant can be seen by comparing figures 8(a) and 8(b). It can readily be seen that allowing the pilot to select the best possible control system time constant can modify the pilot opinion of the flying qualities considerably. In the high-frequency, low-damping region there appears to be a particularly large favorable change of pilot opinion with an increase in the control system time constant from 0.15 second to an optimum value of about 0.7 second. The increased time constant slows down the over-all response of the airplane to stick force inputs. In the high-frequency regions, the pilot considered the rapid response of the airframe objectionable and thus liked to slow down the vehicle response by choosing the slower control system response. In the low-frequency regions, the pilot found that the vehicle response was already too sluggish. Although further reductions in the control system time constant below the minimum value of 0.15 second might effect a slight improvement in over-all response at the lower frequencies, it is probable that greater improvement would be realized if the control power were increased or the stick force per g were decreased.

The control system time constants considered by the pilot to be the optimum are shown in figure 11. The desired values of time constant were not determined in the region between the dashed portion of the $\tau = 0.15$ curve and the curves to the right. However, it was established that in the region to the left of the dashed portion the pilot would like a faster responding control system than that available ($\tau = 0.15$ sec).

The statically stable data of figure 8 are replotted in figure 12 to show the pilot rating as a function of the reciprocal of time to half or double amplitude for constant values of the square of the frequency (k'). Over a fairly large range of this parameter and from a time to double amplitude of about 1 second to a time to half amplitude of about 1 second, there is relatively little effect on pilot opinion of changes A
of frequency. Thus the pilot rating is primarily dependent on damping 4
(which is a function of time to half amplitude in this region). At times 3
to half amplitude of less than 1 second, changes in frequency as well as 8
the damping influence the pilot rating. The boundary between acceptable and unacceptable for normal operations (pilot rating of 5) is at a time to half amplitude of about 1 second over most of the frequency range (it varied from 0.6 at $k' \approx 5$ up to 1.7 at $k' \approx 20$ and back down to 0.75 at $k' \approx 40$).

As was shown earlier, a comparison of figures 12(a) and 12(b) shows that the pilot rating is improved for a given time to half amplitude if the pilot is allowed to select the optimum control system time constant to match the vehicle dynamics.

As the region of oscillatory instability (k' positive, $b' < 0$) is approached, the unacceptable boundary (pilot rating of 6-1/2) is close to the zero damping boundary over most of the frequency range (see figs. 8(a) and 12). Negative damping is unacceptable, and in the very low-frequency and very high-frequency regions a small amount of positive damping is required to remain within the acceptable region. The limit of controllability (8 boundary) is at a value of time to double amplitude of about 1-1/2 seconds over most of the frequency range.

<u>Statically unstable region</u>.- Unlike the oscillatory unstable region, certain portions of the statically unstable region (k' negative) are considered to be acceptable. Figure 10 shows the unacceptable boundary, as well as the controllability boundary, to penetrate well into the statically unstable region. The data are replotted in figure 13 in terms of the time to double amplitude as a function of b'. (In this region T_2 is a function of both k' and b'.) The boundaries seem to be leveling off at the higher values of damping at constant values of time to double amplitude; the unacceptable boundary levels off at a time to double amplitude of about 2/3 second and the limit of controllability at about 1/3 second.

It should be noted that these data were obtained with the pilot paying close attention to his control task, realizing that he was flying an unstable vehicle. While the pilot gave the control task his undivided attention, he found little difficulty in controlling a moderately unstable vehicle ($k' \approx -6$, $b' \approx 4$) and was able to stay near his trim position. However, when the vehicle was allowed to diverge fairly far from trim, the pilot found considerable difficulty in regaining control.

On one flight, when operating with $k' \approx -13$ and $b' \approx 1.5$, the pilot rated the condition 8 to 9, but indicated that he was able to maintain control without too much difficulty. As he was reporting this over the radio, he took a quick glance to look for the chase airplane. This momentary distraction was enough to allow the plane to diverge out of control. This points out the inherent danger of operating an unstable vehicle. A momentary distraction (and pilots do have distractions and cannot devote their undivided attention to the control task) can allow the vehicle to diverge to dangerous attitudes. It might be suspected that these inadvertent attitude excursions may occur more readily at low dynamic pressures where the normal-acceleration cues provided the pilot are very small. However, for situations where the pilot devotes all his attention to controlling an unstable airplane, the results in reference 19 indicate no appreciable effect of a wide variation of normal acceleration per unit angle of attack on the minimum tolerable stability. In any case, it is imperative that sufficient control power be provided to overpower any out-of-trim moment that can be developed at extreme angles of attack.

In determining a desirable control gain with an unstable vehicle, it was necessary to establish a compromise between a relatively insensitive control system which would allow precise control near trim and a more sensitive control system which would give the pilot the feeling that he would not run out of control power if the plane were allowed to diverge a reasonable amount from trim.

Control Techniques

Looking at the pilot's control inputs and the airplane response gives one a feel for the pilot's problems as he attempts to control a vehicle with difficult stability characteristics. Time histories of g, stick force, stick position, and command stabilizer angle are presented in figure 14 for a series of entries into 2g turns. (Command stabilizer angle rather than true stabilizer angle is given because the true stabilizer angle includes the stability augmentation stabilizer motion as well as the motion commanded by the pilot.) In making these records, the pilot was directed to enter a 2g turn and stabilize as rapidly as possible. He was asked to concentrate on the rapid entry and smooth turn rather than on hitting exactly 2g; thus any tendency to level off at other than exactly 2g should be disregarded.

With the good damping and moderate frequency (fig. 14(a)), the entry was rapid with a moderate overshoot. After a few seconds the pilot was able to settle into a smooth turn with almost no oscillations in stick force. This condition was well within the good stability region and was rated 2 – "good, pleasant to fly."

As the damping was decreased (fig. 14(b)), the pilot still made a rapid entry, but there was a continuing oscillation that took almost the full 10 seconds to damp out. This lightly damped condition was rated 4-1/2 – between a rating described as "acceptable, but with unpleasant characteristics" and "unacceptable for normal operation."

As the damping went into the negative region (fig. 14(c)), the pilot changed his control technique completely. He found it necessary to make the turn entry very slowly in order to maintain control. In spite of this gentle approach, there was continued small application of stick force (±1/2 lb), which resulted in an oscillation of about 0.1g. At the end of the 10-second record, the turn had not been established. This condition verged on being uncontrollable. The pilot rated it 8-1/2, between the ratings of "unacceptable, dangerous" and "unacceptable, uncontrollable."

In the statically unstable condition (fig. 14(d)), the pilot again took the gentle approach to the turn entry. There was a continued oscillation of stick force of about ±1 lb and an oscillation in g. At the end of the 10-second record, the turn had not been established. The pilot did not consider this condition quite as bad as the negative damping case, but it was still rated as 8 "unacceptable, dangerous."

There did not seem to be any tendency to revert to pulse-type inputs in stick force as have been reported in other investigations of poor stability characteristics (ref. 8). This is possibly due to the fact that a force-command control was being used and it was not necessary to put in a pulse of stick force (to break through the static friction) to get a small rapid stabilizer response.

Comparison With Other Data

In comparing data from several experiments in this field, it is difficult to find directly comparable conditions because of differences in equipment, in the tasks required of the pilot, in the test conditions, and in the rating systems used. In general, considering these differences, there is very good agreement with other work.

Cornell flight data.- In the tests described in reference 4, a variable stability jet fighter airplane was used with equipment similar to that used in the present investigation. The control system had a constant "equivalent first-order time constant" of about 0.08 second which is slightly faster than the minimum value obtainable with the airplane of the present tests. An average stick force per g of 4.8 lb/g and a stick displacement of 0.09 in./g were used. The test maneuvers were generally similar to those of the present tests.

Figure 15 presents the contours of constant pilot opinion from this work. The best tested boundary was defined as "Acceptable - In this configuration, the airplane's mission could be accomplished reasonably well, but with considerable pilot effort or attention required directly for flying the airplane." This rating seems to be equivalent to a rating of 3-1/2 (table II). The conditions just inside the unacceptable boundary are described as "Acceptable, poor - airplane safe to fly, but pilot effort or attention required is such as to reduce seriously the effectiveness of the airplane in accomplishing its mission." This seems to describe roughly a rating of 5.

Comparisons between figures 8 and 15 show that, in general, there is fairly good agreement. In the higher frequency ranges, the ratings from the present investigation seem to be more generous. The differences in the control system (larger time constant, larger stick force per g, and greater stick displacement for g) tend to give a more favorable rating in this region and are likely to play a large part in the differences in rating.

The available flight data probing the limits of controllability of the longitudinal mode are summarized in figure 16. The 6-1/2 and 8 boundaries of the present report are compared with limit of controllability data from references 4 and 18. Reference 18 presents the results of an investigation in which a twin-engine bomber, modified as a variable-stability airplane, was used to establish the minimum longitudinal stability characteristics with which a safe landing approach could be made.

There seems to be a general agreement between the various sets of data. According to the definitions, the 6-1/2 boundary of the present data would seem to fit between a U_2 and a U_3 of the reference 4 data. The 6-1/2 boundary in figure 16 seems to be slightly more conservative than the reference 4 data with a possible exception in the region of zero k' where the scatter of the reference 4 data, makes a comparison difficult.

In the rating system of the present report, a rating of 7 indicates that it is doubtful that a landing could be made. The agreement with the data of reference 18 is exceptionally good. This is particularly surprising when it is realized that the data of reference 18 were obtained by actually flying landing approaches while the data of the present report were obtained at altitude.

<u>NASA simulator data</u>.- Reference 14 reports an investigation of the aft limit of the maneuver margin using a simulator with freedom of movement in pitch and of vertical translation. In this investigation, it was found that zero maneuver margin was a reasonable aft limit. As zero maneuver margin was reached, the value of b' was quite small. This is in good agreement with the results of the present investigation giving a rating of 6-1/2 or 7 at these conditions. The present data indicate that had there been more damping in the system, the limit of controllability would have been extended into the region of negative maneuver margins.

Reference 15 reports simulator work in which, among other things, a controllability boundary was established with a fixed-based simulator and later checked on the AMAL centrifuge. The boundary established there would agree quite well with a boundary at a pilot rating of 8-1/2 to 9 extrapolated from the data of figure 10.

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Lateral-Directional Controllability Boundaries

A cursory investigation was made of the limits of controllability in the unstable regions of the lateral-directional Dutch-roll mode. The lateral-directional investigation was conducted at 25,000 feet altitude at Mach numbers of 0.6 and 0.75. The variable-stability equipment was adjusted to give the desired natural frequency and the damping was decreased until the limit of controllability was reached. The estimated stability derivatives for the test conditions are shown in table III. The data were taken at a moderately high frequency, at a low frequency, and for a statically unstable condition. In these tests, the rating system of table II was not used; instead the limits of controllability were established as the maximum instability at which the pilot could maintain the angle of sideslip within $\pm 2^{\circ}$ and the angle of bank within $\pm 45^{\circ}$ for 30 seconds. The data obtained are shown in figure 17 along with the comparative data from references 6 and 8.

The values of $|\varphi|/|v_e|$ of the present tests are shown in the figure for the statically stable points (as usually defined, $|\varphi|/|v_e|$ has no meaning in the case of a static divergence). The large differences in $|\varphi|/|v_e|$ at the two frequencies should be noted because of the known effect $|\varphi|/|v_e|$ on pilot rating.

In figure 18 the data of figure 17 have been replotted in terms of the time to double amplitude. At moderately high frequency ($k_D' \approx 20$), the limit of controllability corresponds to a time to double amplitude of about 0.9 second, which is somewhat less than the 1.5 seconds observed in the longitudinal case. At low frequency ($k_D' \approx 2$), the data show a time to double amplitude of the controllability boundary to increase to about 3-1/2 seconds. However, this value is markedly influenced by the large value of $|\varphi|/|v_e|$. For the statically unstable condition, the limit of controllability at zero damping corresponded to a time to double amplitude of 0.5 second. This value compares almost exactly with the limit of controllability found in the longitudinal case for zero damping in the statically unstable region.

Comparison with other data.- Data from references 6 and 8 are included in figures 17 and 18 for reference purposes. In the work reported in reference 6, pilot opinion (based on the rating system of table II) of the Dutch-roll characteristics was obtained in the landing-approach condition using the variable-stability airplane of the present lateral-directional tests. The results of the reference study, representing the extreme limits of controllability (pilot rating of 8), are in fair general agreement with the data of the present study. It is quite possible that the differences in the rating system could account for the small differences seen here.

Reference 8 presents the results of a ground simulator study in which a single-degree-of-freedom yaw chair was used. The boundary presented by the long curve in figures 17 and 18 represents the conditions at which the pilot lost control of the chair. There is good agreement with the present results at the moderate frequency ($k_D' \approx 20$), but the agreement becomes progressively poorer as k_D' is reduced toward zero. Results in reference 8 also indicated that a static divergence corresponding to a time to double amplitude of 1/3 second was marginally controllable. Unfortunately the degree of damping associated with this figure is not given. This value of time to double amplitude does not agree too well with the 0.5 second found in the present lateral-directional investigation for zero damping in the statically unstable condition. However, it does agree almost exactly with the value found in the longitudinal portion at higher values of damping (fig. 13).

The agreement in the times to double amplitude of the limit of controllability between the lateral-directional and the longitudinal tests is surprising. It makes one wonder if this limit of controllability is a function of the human reaction time only and independent of the mode of operation. In this regard reference 22 has indicated the reaction times to be about the same for conditions of initially large longitudinal or lateral disturbances. Tests with more directly comparable rating systems would be interesting in this respect.

Combined Longitudinal and Lateral-Directional Dynamics

The present tests have treated the longitudinal and lateral-directional modes of motion separately. In the flight tests, the mode of motion not under study was stable and generally well within what the pilot considered satisfactory regions of controllability. In the simulator tests discussed in comparison with the flight work, there was freedom of motion only in the mode under study. There is justification for this approach, in addition to its simplicity, in that a good possibility exists that only one channel of stability augmentation will fail at any one time. However, it is also very desirable to consider the situation where all modes of the vehicle behavior deteriorate to marginal values simultaneously.

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Reference 17 demonstrates that the pilots find it much more difficult to control a vehicle in which the stability characteristics deteriorate in both the lateral-directional modes at the same time than when only one has poor stability characteristics. Figure 19 taken from reference 17 shows those effects. Here, when a longitudinal mode with a rating of 4 is combined with a lateral-directional mode of rating 6, the vehicle becomes virtually unflyable with a rating of 9. Thus, we can expect the ratings presented earlier to be very optimistic should the stability deteriorate in both modes simultaneously.

CONCLUDING REMARKS

The pilot opinion of the flying qualities of vehicles covering a wide range of longitudinal dynamic characteristics has been determined by the use of a variable-stability airplane. Particular emphasis has been placed on the regions of dynamic and static instability.

There was considerable pilot learning associated with the regions of poor stability. After a period of operation at minimum levels of stability, the pilots tended to be more lenient in their ratings than they were on first encountering the poor stability characteristics.

In the statically stable region, the pilots considered a time to damp to half amplitude of the longitudinal oscillation of about 1 second to be the maximum acceptable for normal operation. For emergency conditions, they would accept values of damping close to zero for most of the frequency range. The extremes of controllability correspond to time to double amplitude of the oscillation of about 1-1/2 seconds.

In the statically unstable region the boundary for emergency operation leveled off at a time to double amplitude of about 2/3 second at the higher values of damping and were somewhat higher close to zero damping. The limit of controllability was at a time to double amplitude of about 1/3 second at the higher values of damping.

In a very brief study, the limits of controllability in the lateral-directional Dutch-roll mode were found to be in fairly close agreement with those of the longitudinal mode.

In handling vehicles which are moderately statically unstable, it was found that the pilot had relatively little trouble in maintaining control as long as he gave his undivided attention to the problem. The danger of the situation is that when the pilot's attention is distracted for a period of time, the vehicle can diverge to large deflections before the pilot notices the pitching or normal accelerations associated with the divergence. In such cases, it is imperative that sufficient control be provided to overcome any out of trim moment that may occur for large excursions of the vehicle from trim.

The above conclusions were reached in separate investigations of the longitudinal and the lateral-directional modes. A previous simulator study indicates that should the limits of controllability be approached in both modes simultaneously, the pilot opinion of the vehicle's flying characteristics would deteriorate well below that for either mode separately. Thus the boundaries indicated herein would undoubtedly be very optimistic should the limits of controllability be approached simultaneously in both the longitudinal and lateral-directional modes.

In general, the results of this investigation agree quite well with other work done in this field.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Jan. 12, 1961

APPENDIX A

DETERMINATION OF ROOTS FOR STATICALLY DIVERGENT FLIGHT CONDITIONS

In the analysis of flight records where there is a static divergence and positive damping, the constant term of the characteristic equation is negative giving rise to the following situation:

$$\frac{n}{\delta} = \frac{C_{o_n}}{s^2 + b's + k'} = \frac{C_{o_n}}{(s + \sigma_1)(s - \sigma_2)}$$

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where

$$b' = \sigma_1 - \sigma_2$$

$$k' = -\sigma_1\sigma_2$$

The time response to a control impulse for this situation will be

$$n(t) = C_{o_n}\left(\frac{e^{\sigma_2 t} - e^{-\sigma_1 t}}{\sigma_1 + \sigma_2}\right)$$

This response is plotted in figure 20 on semilog paper for two values of the stable root. It is noted that each term is a straight line and that the total response approaches asymptotically that of the unstable term at large values of time as the contribution of the stable term becomes small.

Then given a record of the response to an impulse, the roots can be extracted by the following steps.

1. Plot the response on semilog paper as a function of time.

2. Draw the asymptote to this curve.

3. Plot the difference between the time history and the asymptote.

4. Draw the asymptote to this second curve. (This curve will be a straight line if the original curve had only two roots.)

5. The slopes of these lines give the value of the roots, that is, a negative slope is associated with the stable root, σ_1 and a positive slope with the unstable root σ_2. The pertinent relationships are:

$$\sigma_1 = -\frac{1}{\Delta t}\, \ln \left(\frac{n_{t=t_2}}{n_{t=t_1}}\right)$$

$$\sigma_2 = \frac{1}{\Delta t}\, \ln \left(\frac{n_{t=t_2}}{n_{t=t_1}}\right)$$

This analysis works well if the stable root is large enough so that its response essentially goes to zero within the time of the record sample and if there is not too much response unrelated to the two roots. A similar technique can be used to separate two stable roots if they are not too close to the same value.

REFERENCES

1. McFadden, Norman M., Pauli, Frank A., and Heinle, Donovan R.: A Flight Study of Longitudinal-Control-System Dynamic Characteristics by the Use of a Variable-Control-System Airplane. NACA RM A57L10, 1958.

2. Sadoff, Melvin: The Effects of Longitudinal Control-System Dynamics on Pilot Opinion and Response Characteristics as Determined From Flight Tests and From Ground Simulator Studies. NASA MEMO 10-1-58A, 1958.

3. Harper, Robert P., Jr.: Flight Evaluations of Various Longitudinal Handling Qualities in a Variable-Stability Jet Fighter. WADC Tech. Rep. 55-299, July 1955.

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4. Chalk, Charles R.: Additional Flight Evaluations of Various Longitudinal Handling Qualities in a Variable-Stability Jet Fighter. WADC Tech. Rep. 57-719, pt. II, 1958.

5. Newell, Fred: Effects of Breakout Force on Longitudinal Handling Qualities. WADC Tech. Rep. 57-155, Dec. 1957.

6. McNeill, Walter E., and Vomaske, Richard F.: A Flight Investigation to Determine the Lateral Oscillatory Damping Acceptable for an Airplane in the Landing Approach. NASA MEMO 12-10-58A, 1959.

7. Liddell, Charles J., Jr., Creer, Brent Y., and Van Dyke, Rudolph D., Jr.: A Flight Study of Requirements for Satisfactory Lateral Oscillatory Characteristics of Fighter Aircraft. NACA RM A51E16, 1951.

8. Cheatham, Donald C.: A Study of the Characteristics of Human-Pilot Control Response to Simulated Aircraft Lateral Motions. NACA Rep. 1197, 1954.

9. Russell, Lindsay: Characteristics of the Human as a Linear Servo-Element. Dept. of Electrical Engineering, M.I.T., 1951.

10. McRuer, Duane T., and Krendel, Ira S.: Dynamic Response of Human Operators. WADC TR 56-524, Oct. 1957.

11. Ashkenas, Irving L., and McRuer, Duane T.: The Determination of Lateral Handling Quality Requirements From Airframe-Human Pilot System Studies. WADC TR 56-135, June 1959.

12. Hall, Ian A. N.: Effects of Controlled Element on the Human Pilot. WADC TR 57-509, Aug. 1958.

13. Moul, Martin T., and Brown, Lawrence W.: Effect of Artificial Pitch Damping on the Longitudinal and Rolling Stability of Aircraft With Negative Static Margins. NASA MEMO 5-5-59L, 1959.

14. Brown, B., Porter, and Johnson, Harold I.: Moving Cockpit Simulator Investigation of the Minimum Tolerable Longitudinal Maneuvering Stability. NASA TN D-26, 1959.

15. Holleman, Euclid C., Armstrong, Neil A., and Andrews, William H.: Utilization of the Pilot in the Launch and Injection of a Multi-stage Orbital Vehicle. Inst. Aero. Sci. Preprint 60-16, 1960.

16. Sadoff, Melvin, McFadden, Norman M., and Heinle, Donovan R.: A Study of Longitudinal Control Problems at Low and Negative Damping and Stability, With Emphasis on Effect of Motion Cues. NASA TN D-348, 1961.

17. Creer, Brent Y., Heinle, Donovan R., and Wingrove, Rodney C.: Study of Stability and Control Characteristics of Atmosphere-Entry Type Aircraft Through Use of Piloted Simulators. IAS Paper No. 59-129.

18. Bull, Gifford: Minimum Flyable Longitudinal Handling Qualities of Airplanes. Cornell Lab. Inc. Rep. No. TB-1313-F-1, Dec. 1959.

19. Brissenden, Roy F., Alford, William L., and Mallick, Donald L.: Flight Investigation of Pilot's Ability to Control an Airplane Having Positive and Negative Static Longitudinal Stability Coupled With Various Effective Lift-Curve Slopes. NASA TN D-211, 1960.

20. Cooper, George E.: Understanding and Interpreting Pilot Opinion. Aero. Eng. Rev., vol. 16, no. 3, March 1957, pp. 47-51, 56.

21. Draper, C. S., McKay, W., and Lees, S.: Instrument Engineering, vol. III. McGraw Hill Book Co., Inc., Sept. 1952.

22. Kuehnel, Helmut A.: In-Flight Measurements of the Time Required for a Pilot to Respond to an Aircraft Disturbance. NASA TN D-221, 1960.

TABLE I.- BASIC STABILITY DERIVATIVES FOR THE LONGITUDINAL TEST AIRPLANE
[Test flight condition, Mach number 0.80, altitude 35,000 feet]

M_δ	-625,000 ft-lb/radian	S	278.9 sq ft
Z_δ	-39,800 lb/radian	$\bar{c}$	8.08 ft
M_α	-395,000 ft-lb/radian	V	779 ft/sec
Z_α	-389,000 lb/radian	$\frac{\rho V^2}{2}$	224 lb/ft^2
M_q	-12,600 ft-lb/radian/sec	C_{O_n}	19,548
$M_{\dot{\alpha}}$	-8,000 ft-lb/radian/sec	C_{O_q}	-25.1
I_y	26,200 slug-ft^2	C_{O_α}	-23.9
m	444 slugs	C_{1_q}	-23.8

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TABLE II.- PILOT OPINION RATING SYSTEM

	Adjective rating	Numerical rating	Description	Primary mission accomplished	Can be landed
Normal operation	Satisfactory	1	Excellent, includes optimum	Yes	Yes
		2	Good, pleasant to fly	Yes	Yes
		3	Satisfactory, but with some mildly unpleasant characteristics	Yes	Yes
Emergency operation	Unsatisfactory	4	Acceptable, but with unpleasant characteristics	Yes	Yes
		5	Unacceptable for normal operation	Doubtful	Yes
		6	Acceptable for emergency condition only[1]	Doubtful	Yes
No operation	Unacceptable	7	Unacceptable even for emergency condition[1]	No	Doubtful
		8	Unacceptable - dangerous	No	No
		9	Unacceptable - uncontrollable	No	No
	Catastrophic	10	Motions possibly violent enough to prevent pilot escape	No	No

[1]Failure of a stability augmenter

TABLE III.- DATA FOR CONDITIONS OF MINIMUM

LATERAL-DIRECTIONAL CONTROLLABILITY

M	k_D'	b_D'	Stability derivatives								$\frac{\lvert\varphi\rvert}{\lvert v_e\rvert}$
			N_p	N_{δ_a}	N_{δ_r}	L_β	L_r	L_p	L_{δ_a}	L_{δ_r}	
0.75	18.9	-1.57	0.03	-1.56	-9.2	-76.5	-0.11	-2.44	-31.5	18.6	0.26
.75	1.6	-.40	-.01	-1.56	-9.2	-63.9	-.65	-2.44	-31.5	18.6	2.20
.60	-2.0	0	-.02	-1.36	-5.8	-15.5	2.33	-2.15	-24.2	21.9	- - -

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M	I_x	I_z	I_{xz}
0.60	7,340	23,370	-20
.75	7,340	23,380	320

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Figure 1.- Ames variable-control-system, variable-stability YF-86D.

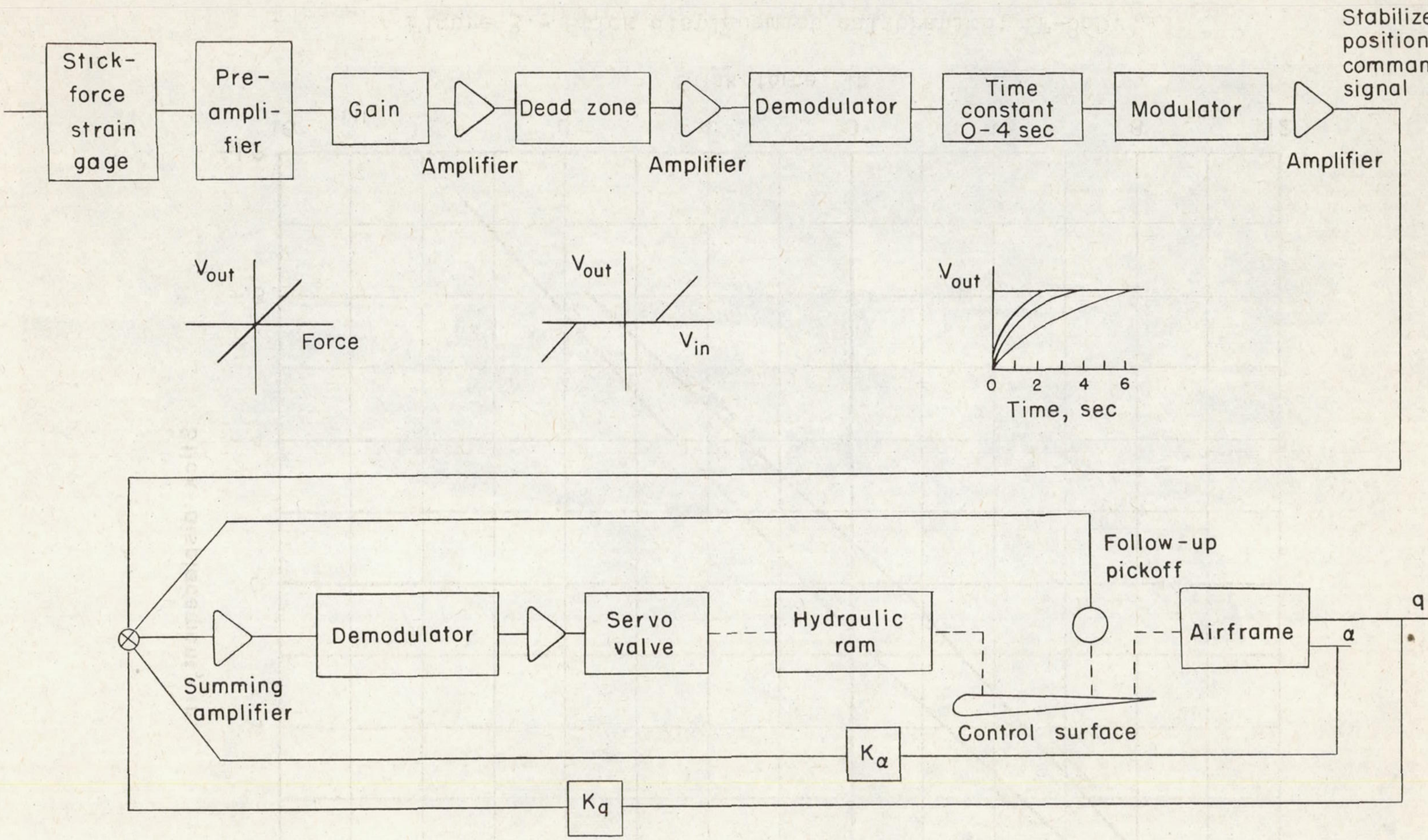

Figure 2.- Functional block diagram of control system; Ames variable-control-system, variable-stability YF-86D.

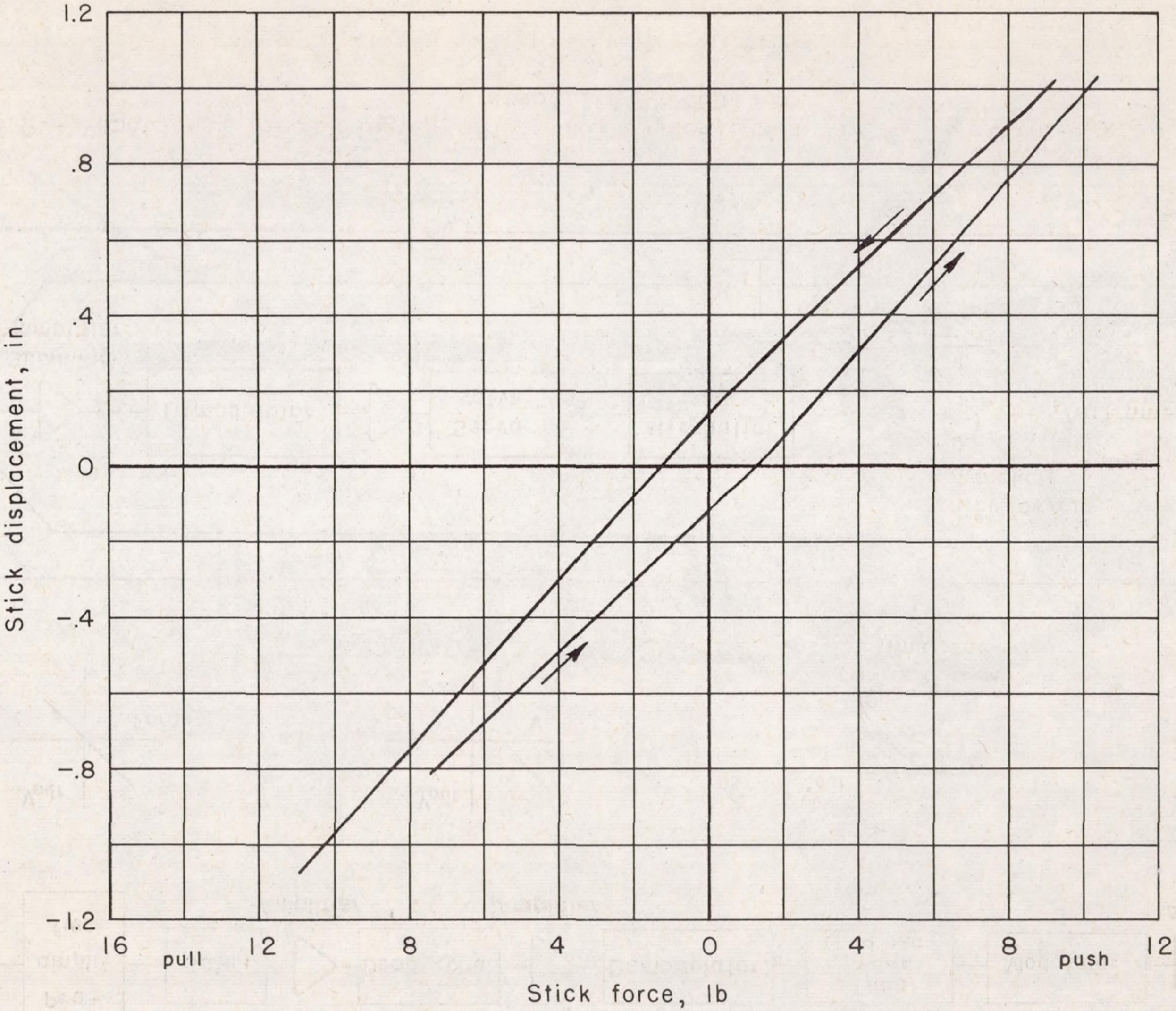

Figure 3.- Stick displacement calibration; YF-86D.

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Figure 4.- Ames variable-stability F-86E.

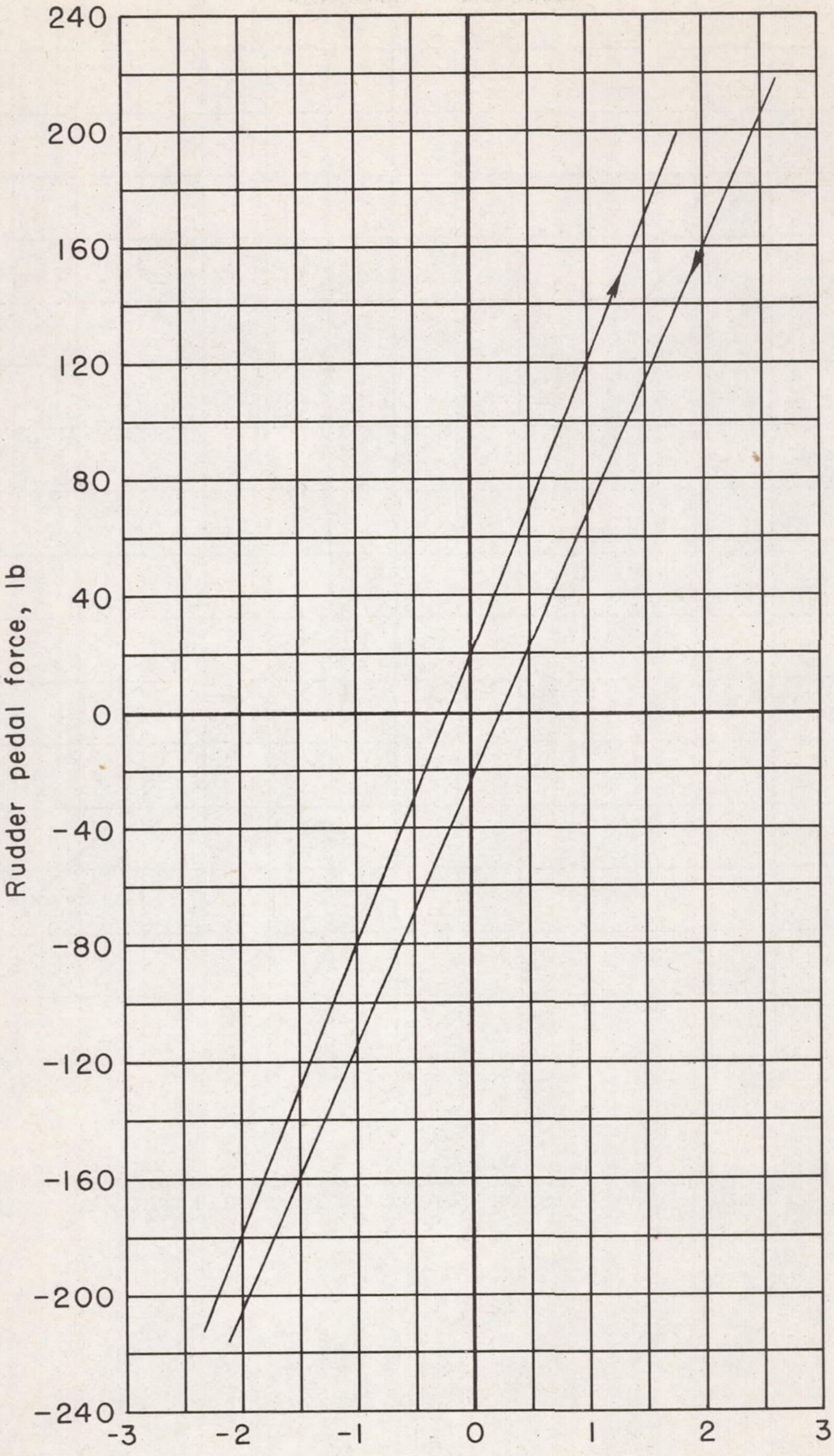

Figure 5.- Rudder pedal control force calibration.

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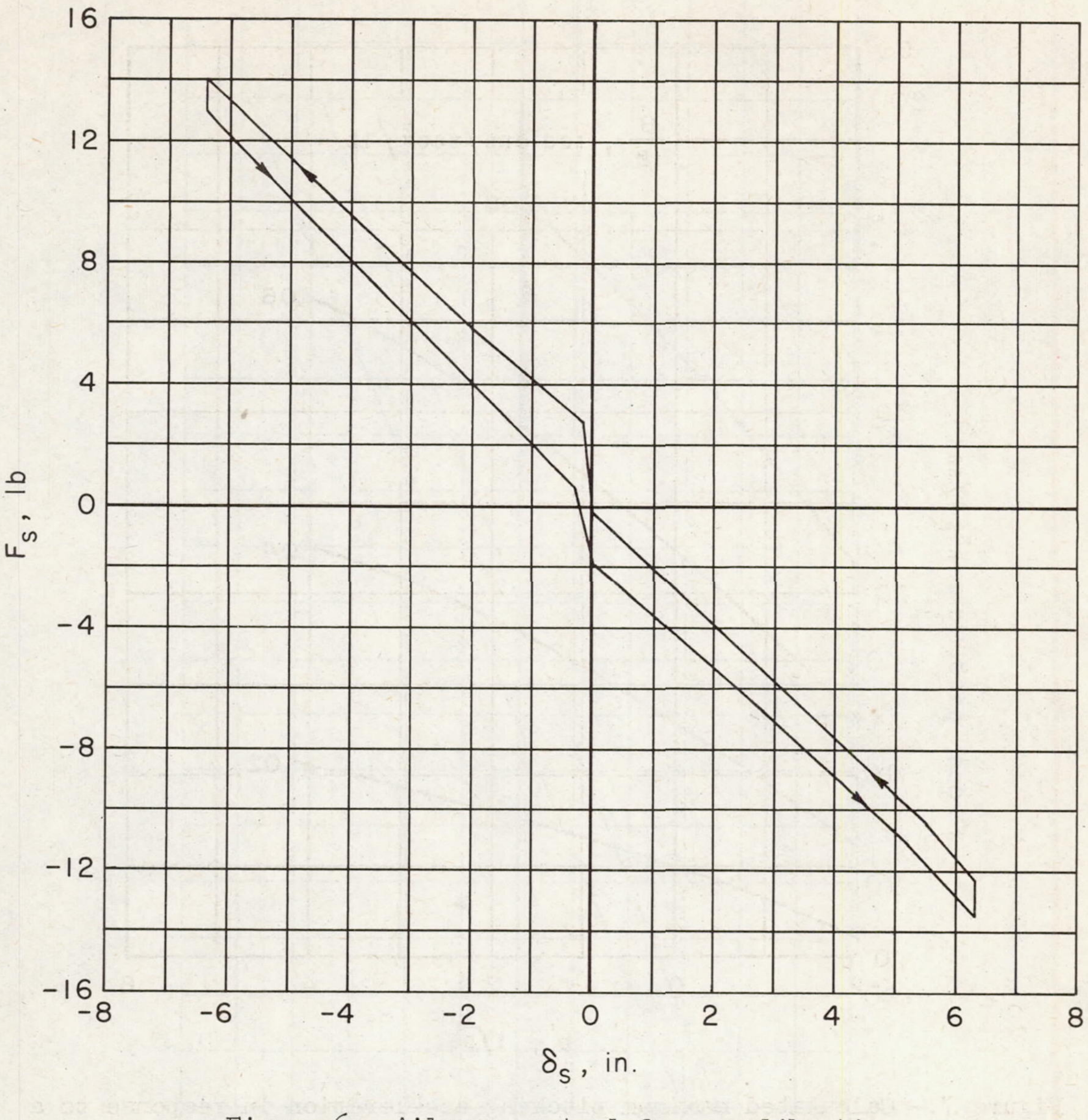

Figure 6.- Aileron control force calibration.

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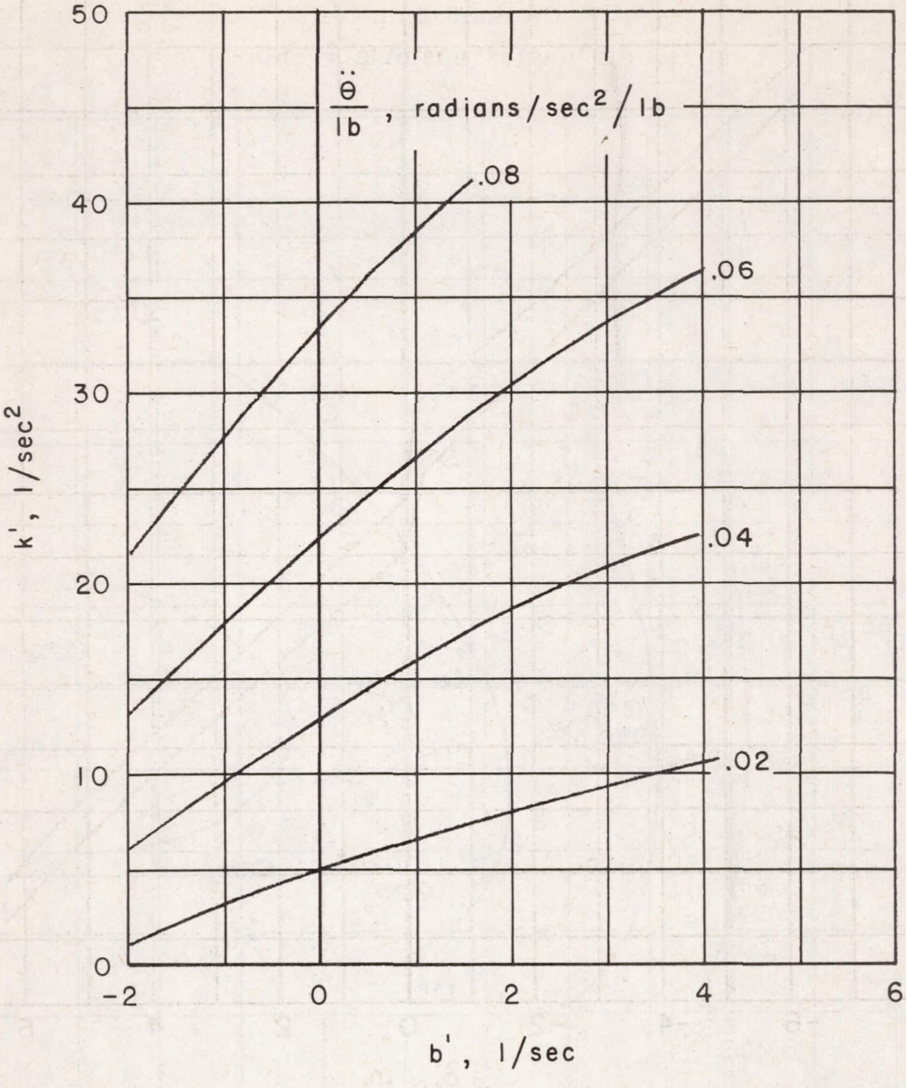
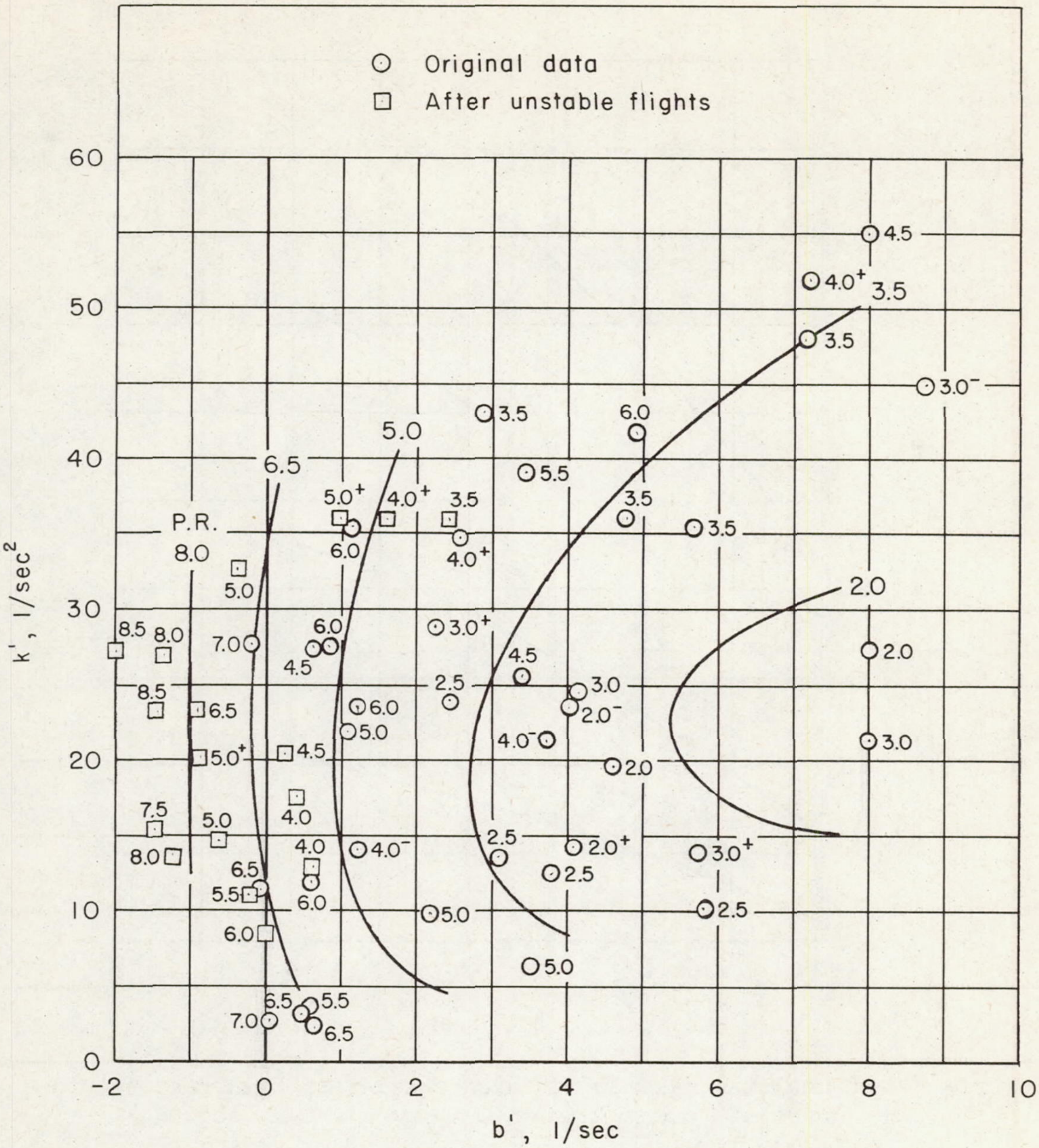


Figure 7.- Calculated maximum pitching acceleration in response to a step in stick force; $F_s/g = 10$ lb/g; $\tau = 0.15$ second.



(a) Control system time constant, 0.15 sec.

Figure 8.- Contours of constant pilot opinion; $F_s/g = 10$ lb/g.

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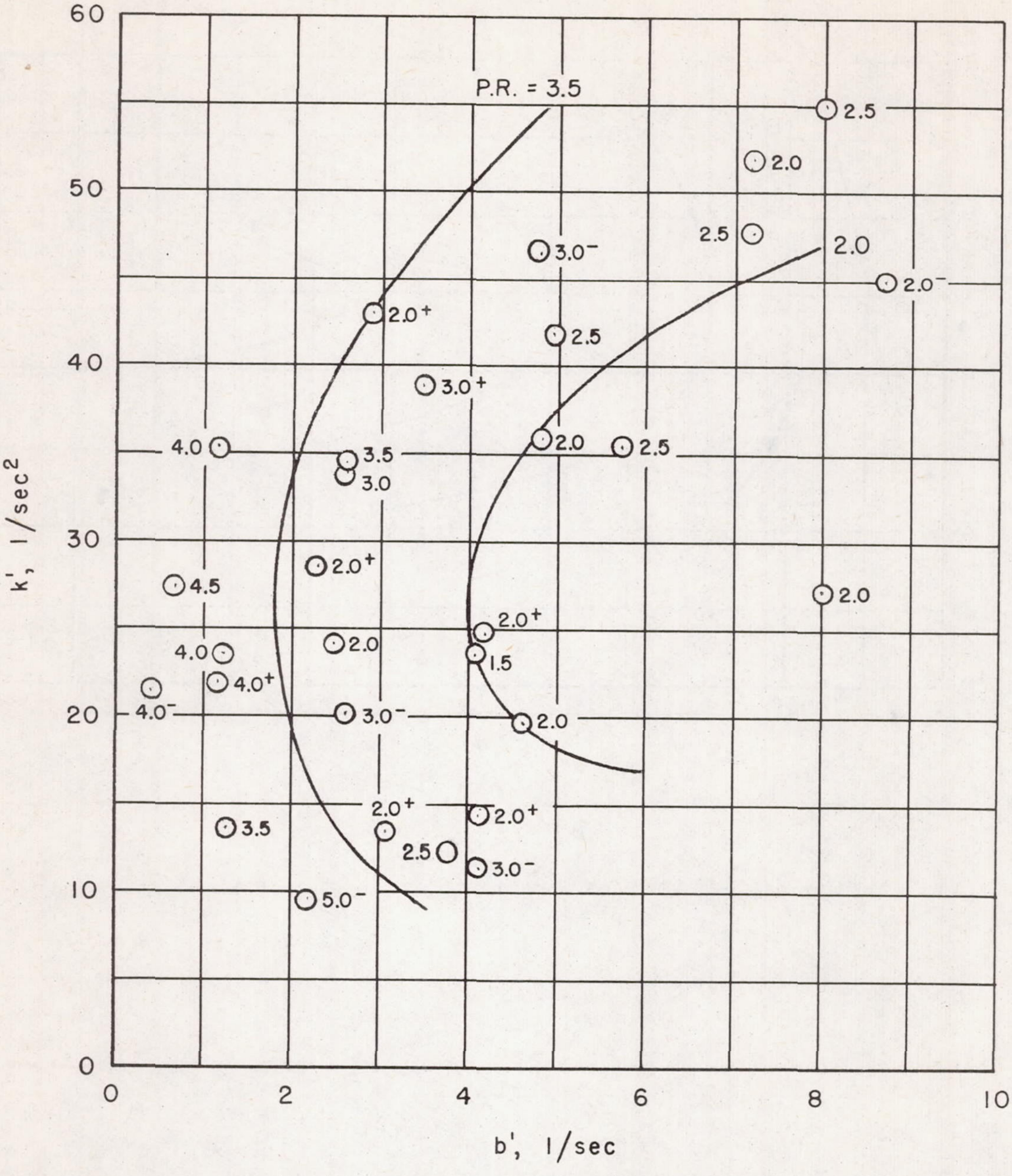
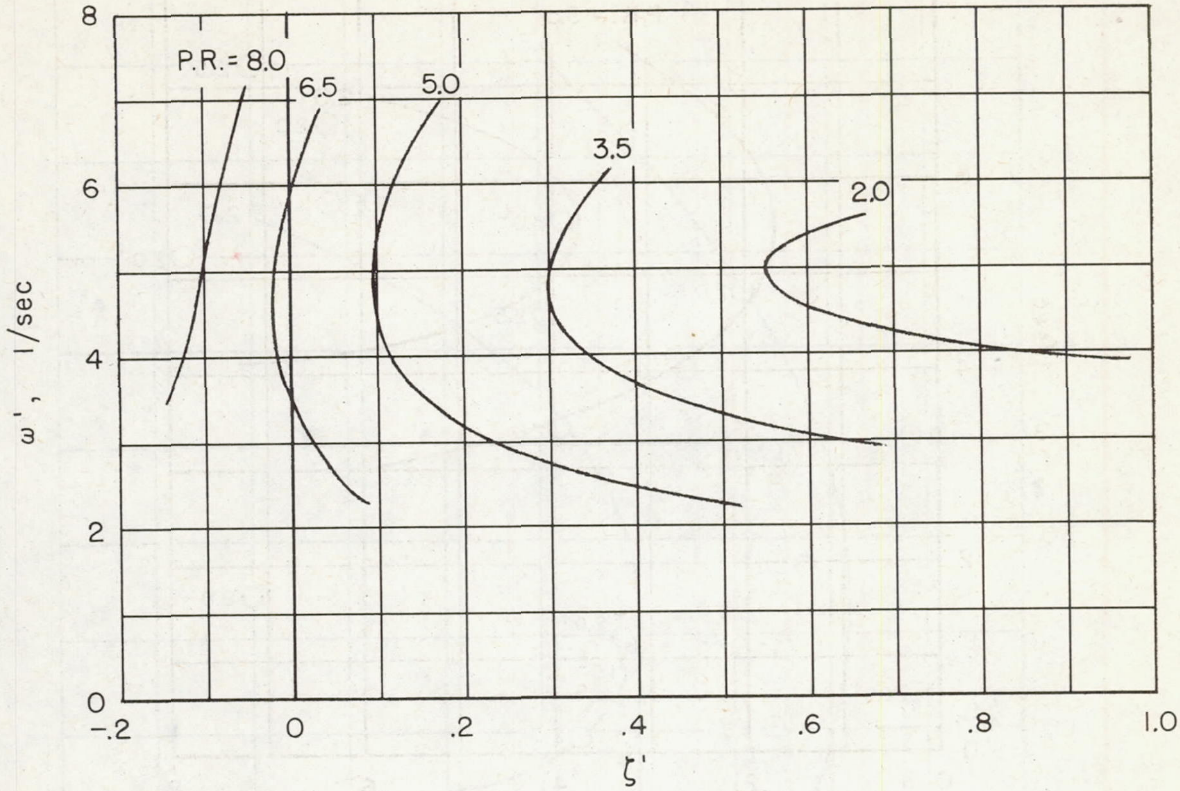


(b) Optimum control system time constants.

Figure 8.- Concluded.



(a) τ = 0.15 sec.

Figure 9.- Pilot rating contours - longitudinal mode.

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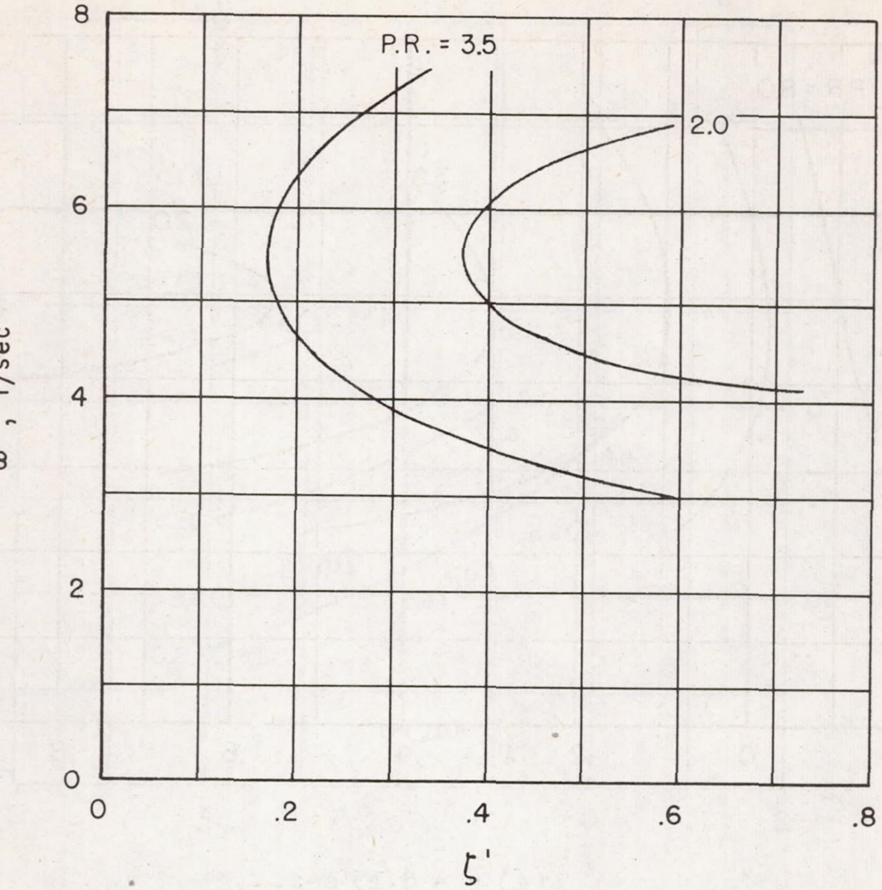

(b) Optimum τ.

Figure 9.- Concluded.

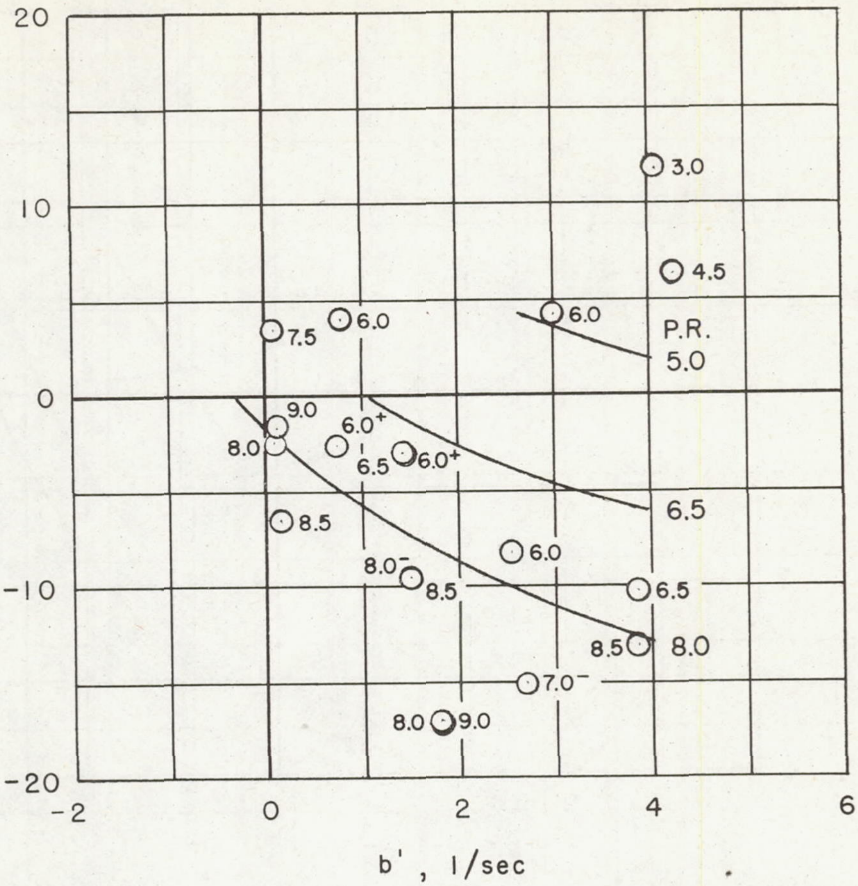

Figure 10.- Contours of constant pilot opinion in statically unstable region; constant stick-to-stabilizer gearing.

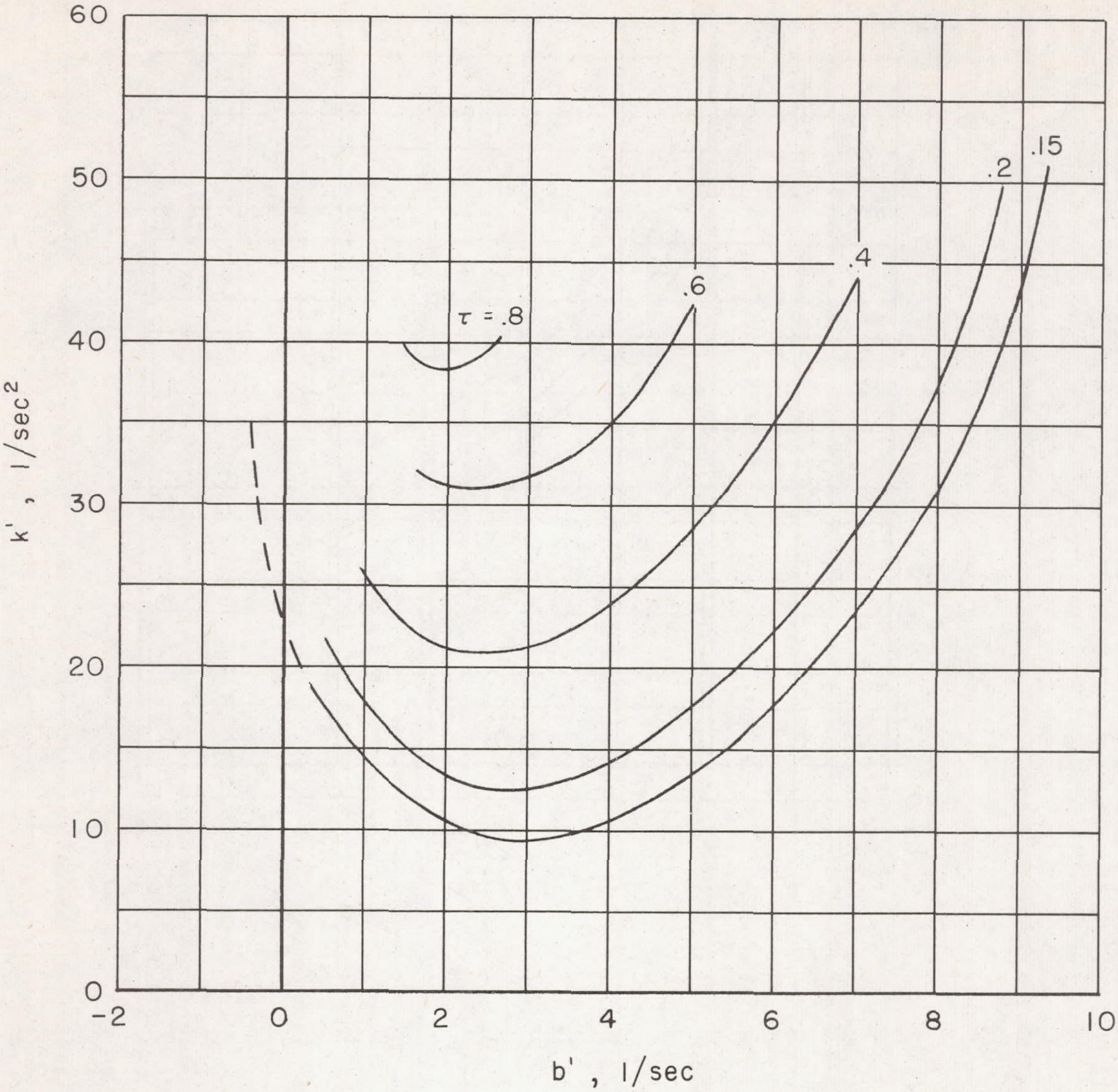

Figure 11.- Control system time constants selected as best by the pilot.

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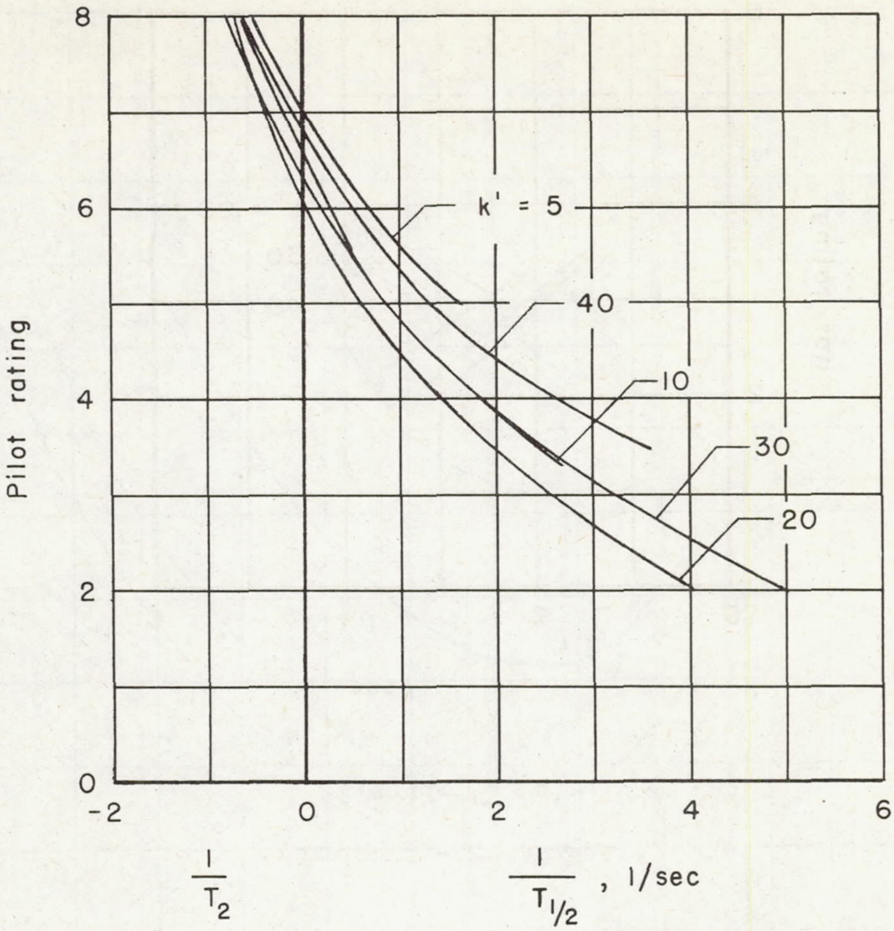
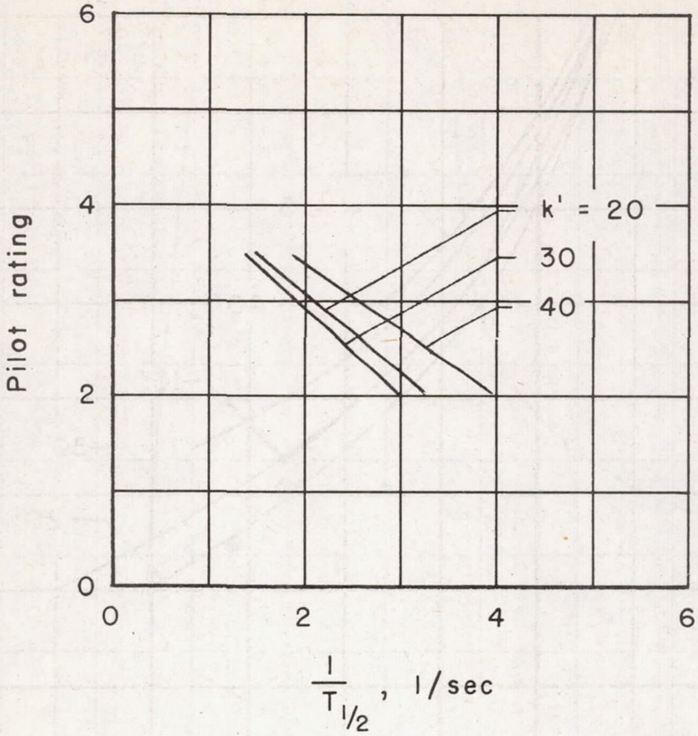


(a) τ = 0.15 sec.

Figure 12.- Pilot rating as a function of $1/T_2$ and $1/T_{1/2}$ for constant values of k'.



(b) τ, optimum.

Figure 12.- Concluded.

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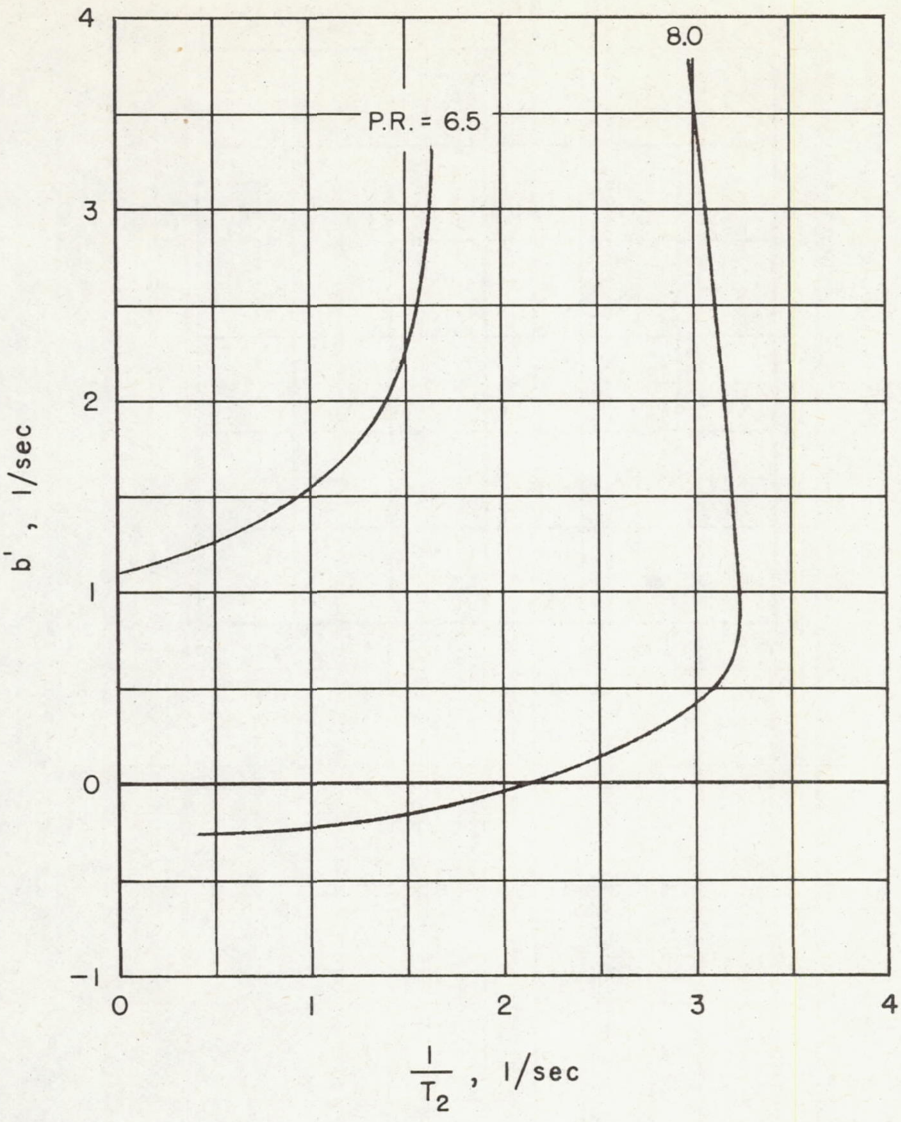

Figure 13.- Contours of constant pilot opinion in the statically unstable region.

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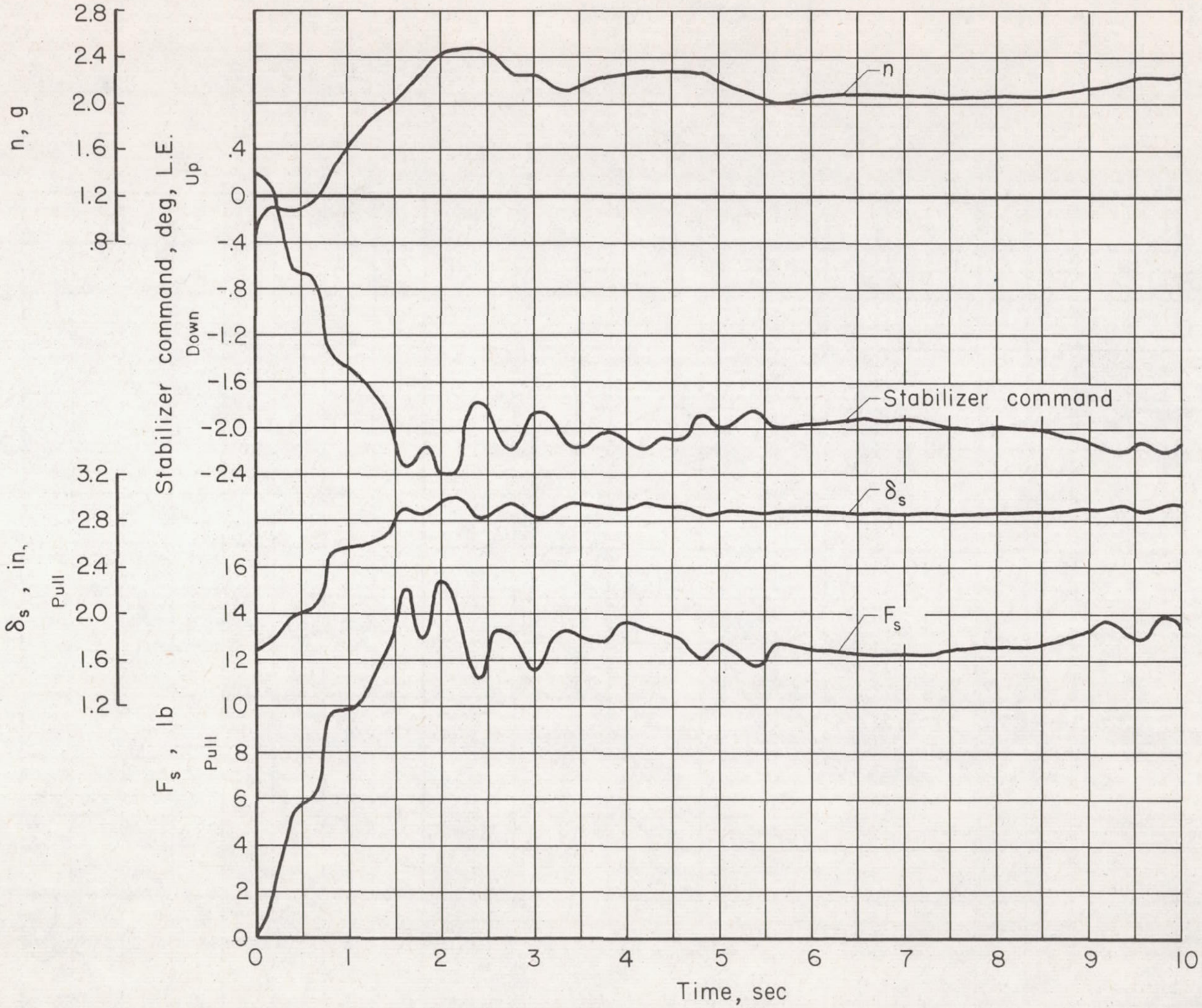
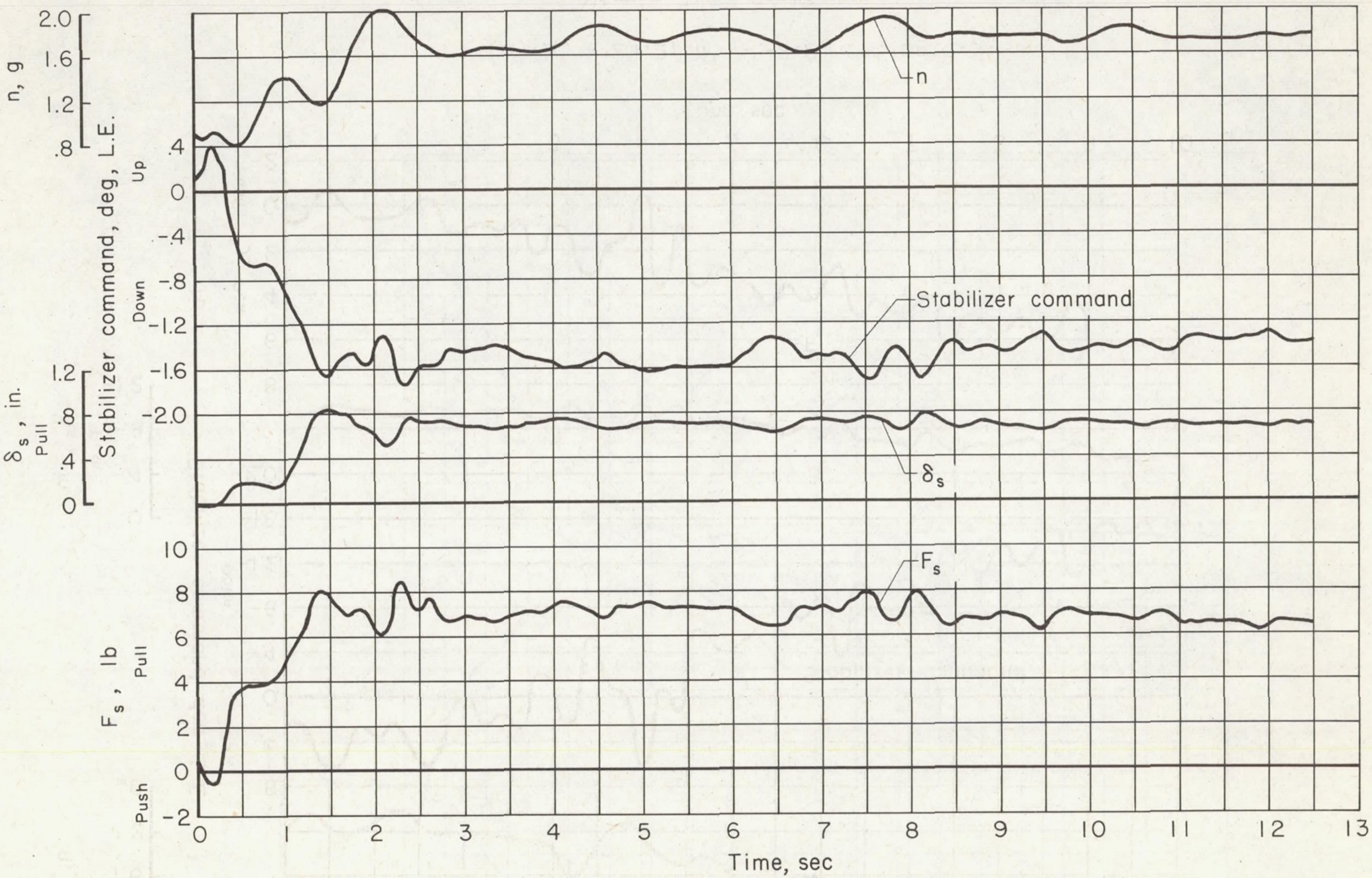


(a) $k' = 26.01$; $b' = 6.12$; rating 2.

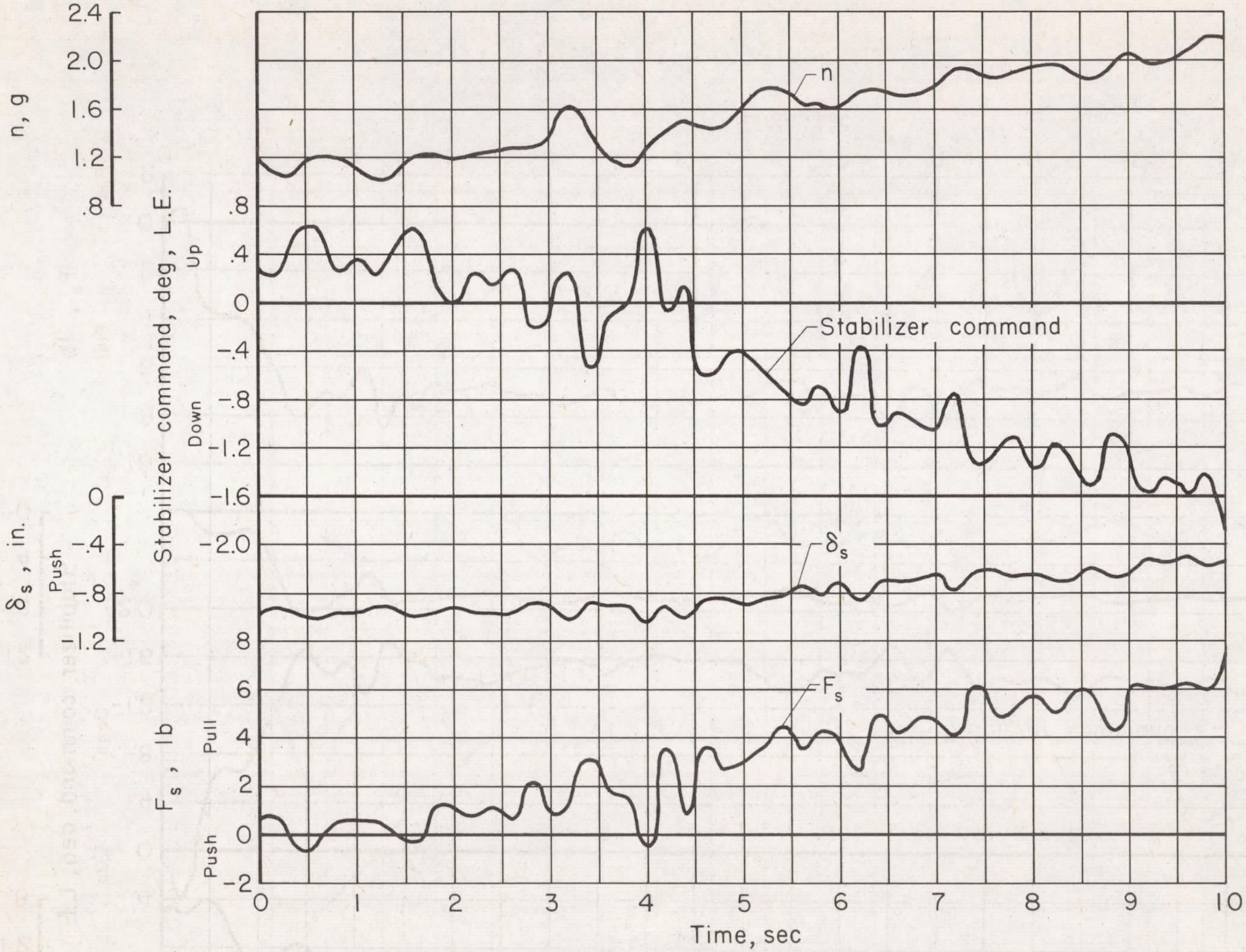
Figure 14.- Time history of entry into a 2g turn.



(b) k' = 27.04; b' = 0.73; rating = 4.5.

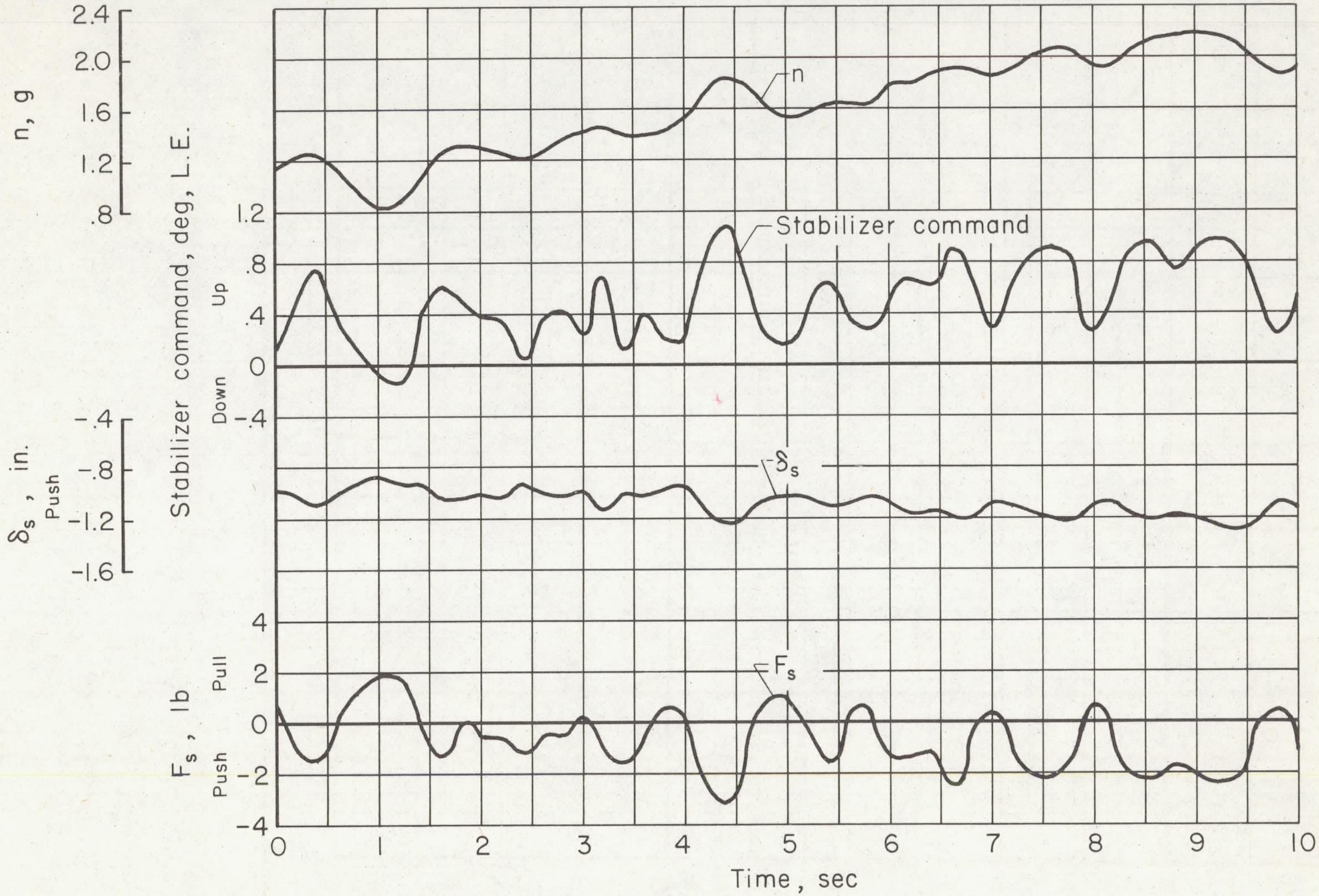
Figure 14.- Continued.

A 4 3 8



(c) k' = 26.2; b' = -2.2; rating 8.5.

Figure 14.- Continued.



(d) k' = -2.00; b' = 0.1; rating 8.0.

Figure 14.- Concluded.

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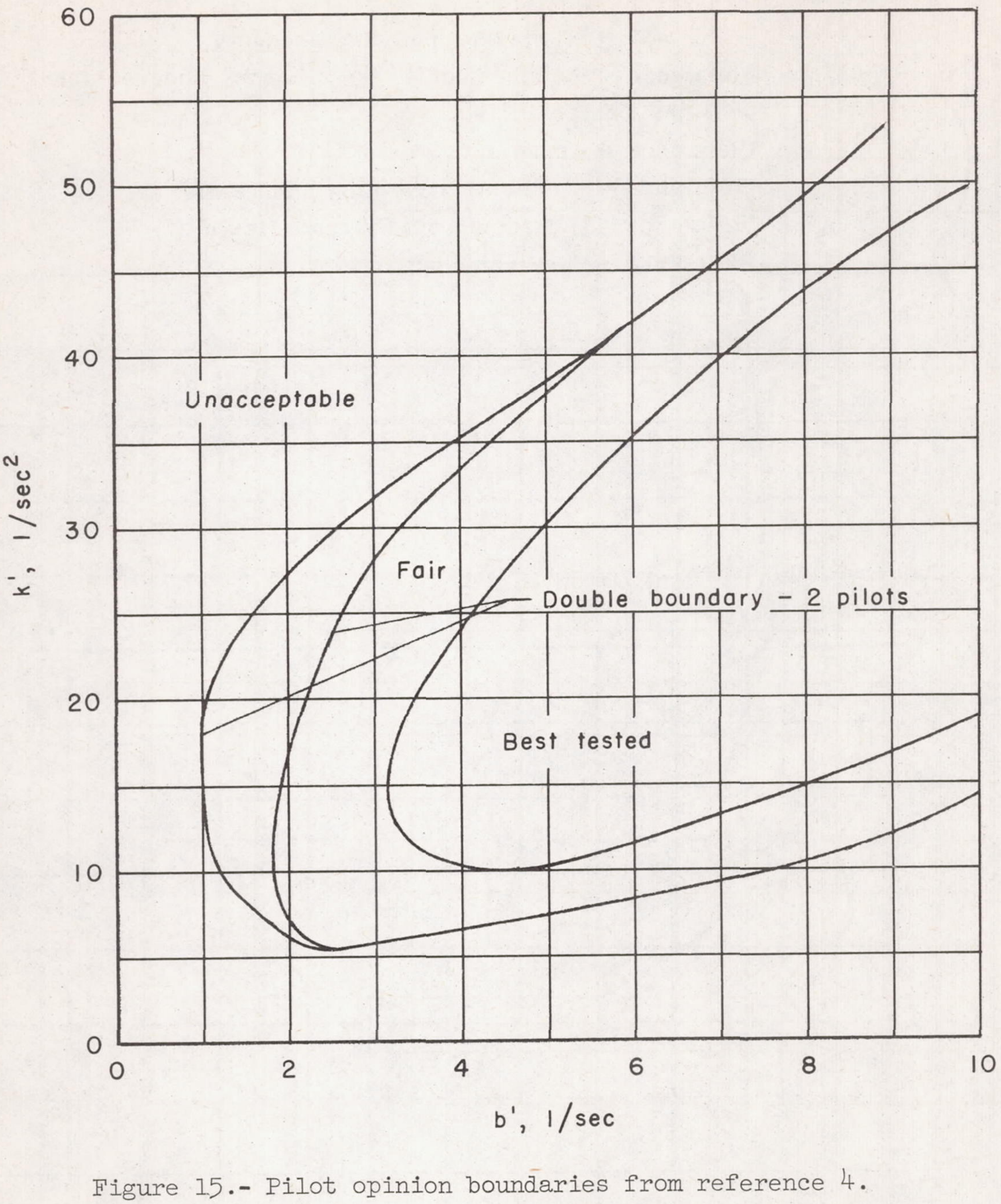

Figure 15.- Pilot opinion boundaries from reference 4.

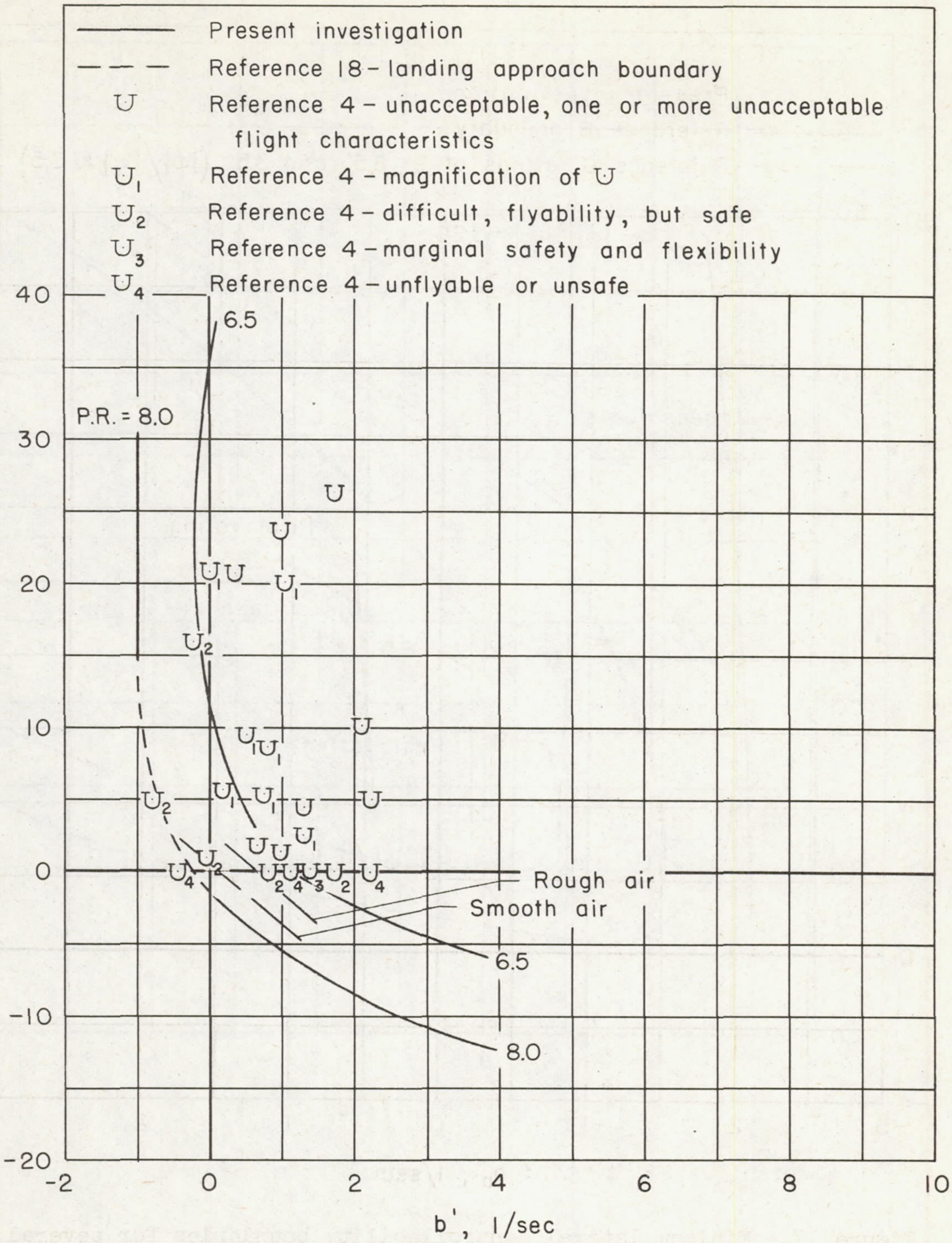

Figure 16.- Summary of data obtained in flight establishing the limits of controllability of the longitudinal mode.

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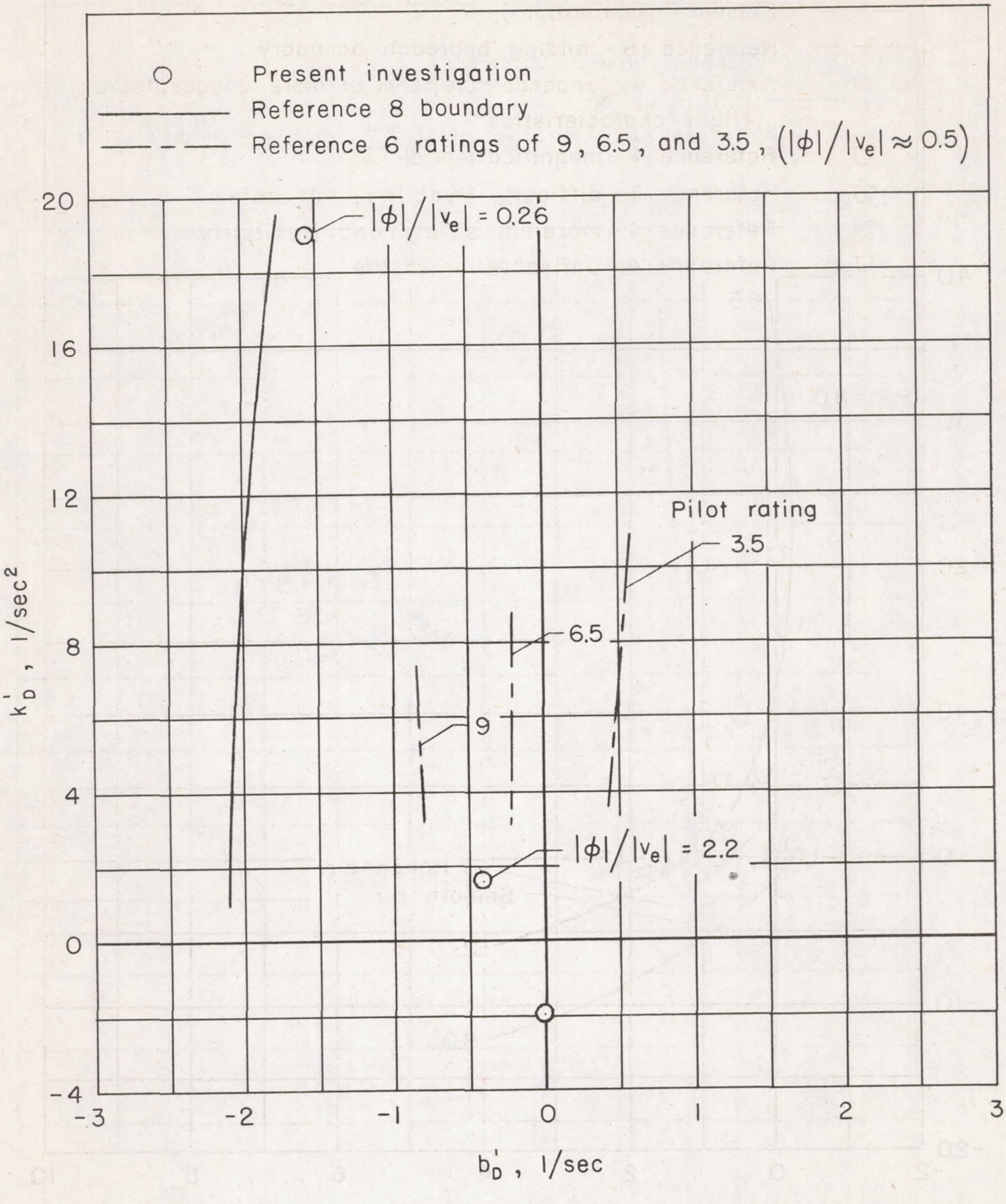

Figure 17.- Minimum lateral controllability boundaries for several investigations.

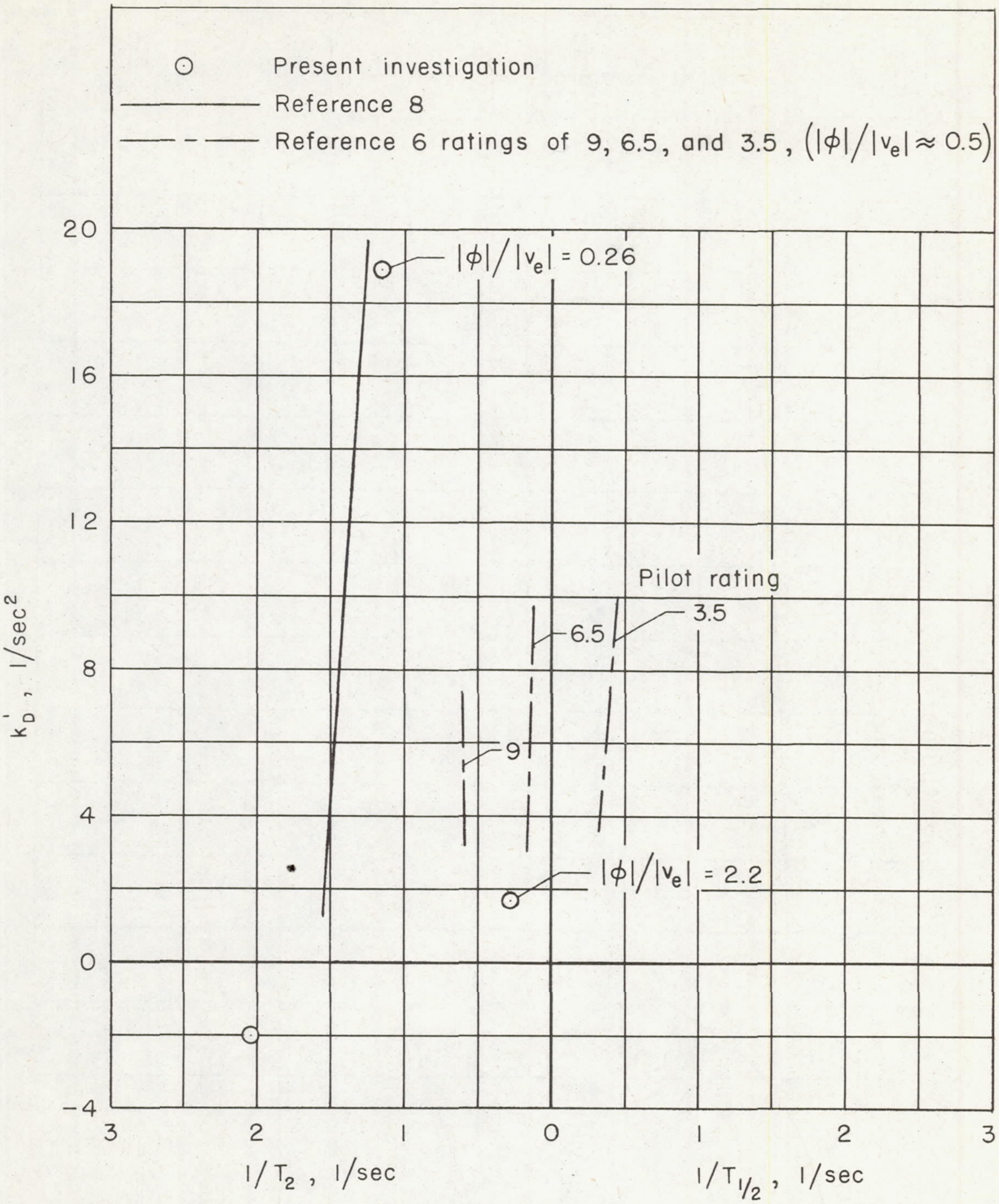

Figure 18.- Limits of controllability as a function of time to double amplitude.

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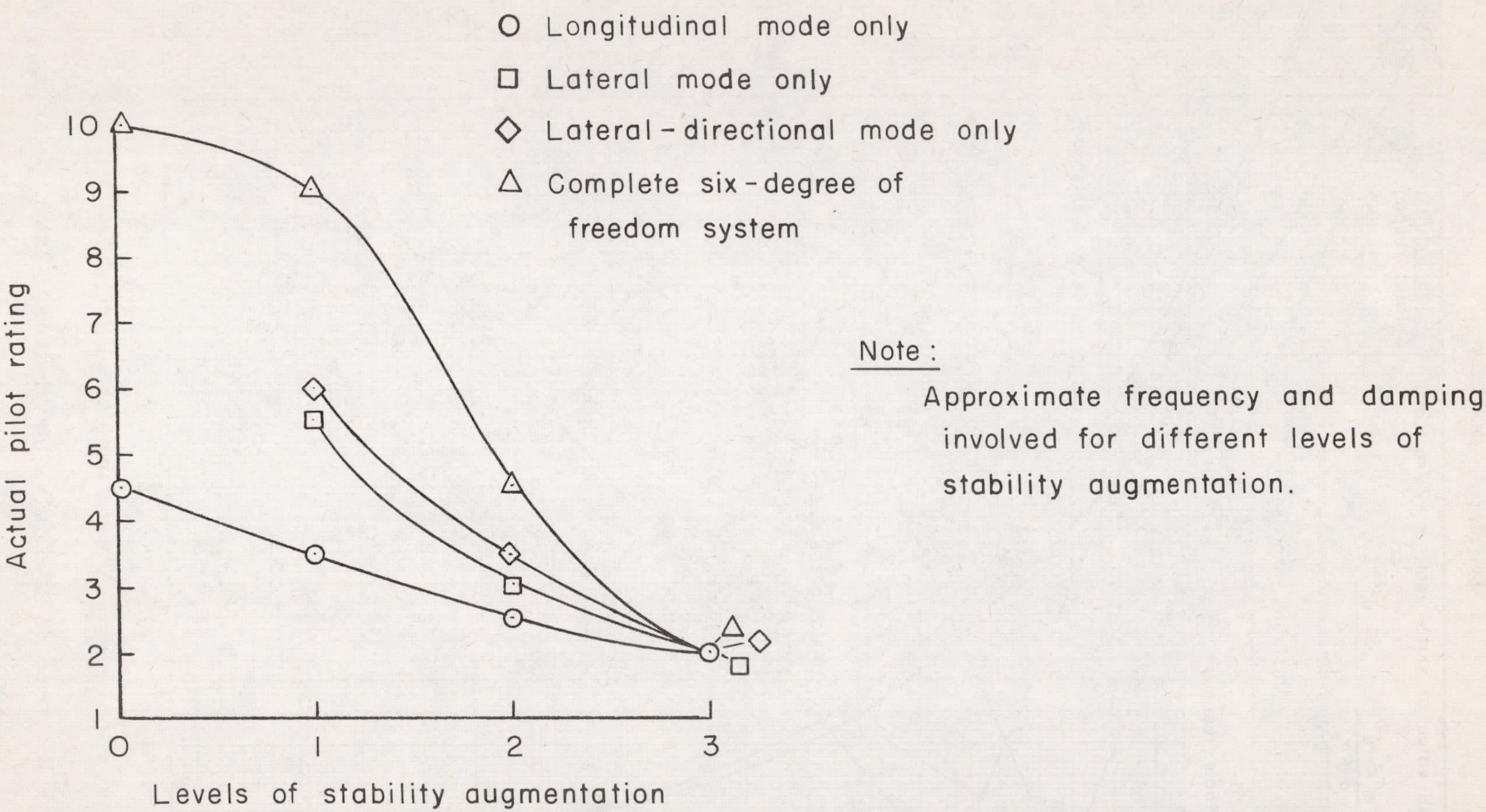

Figure 19.- Comparison of pilot opinions obtained from flying separate modes of airframe motion with pilot opinion obtained from flying complete six-degrees-of-freedom system (ref. 17).

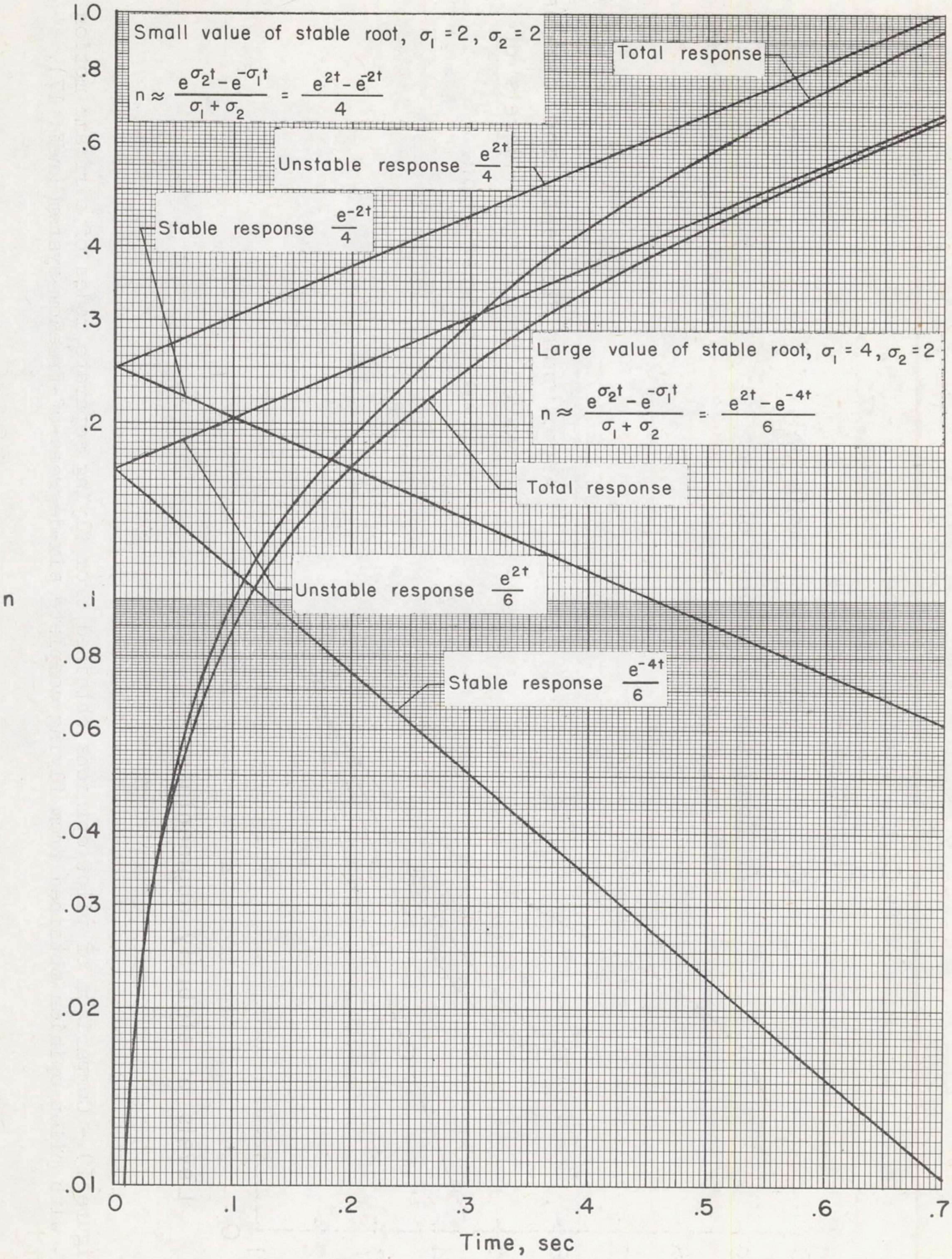

Figure 20.- Response of unstable second-order system to an impulse.